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**Highlights (for review)**

- We present an Improved Eigenvector-based Centrality Measure (IECM) for temporal networks
- The coupling strength between proximity layers is calculated as the inter-layer similarity.
- We introduce the TOPSIS method to measure the global performance.
- The inter-layer coupling strength plays an important role for identifying the influential nodes.

# Inter-layer similarity-based eigenvector centrality measures for temporal networks

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**Abstract**

Identifying the influential nodes in temporal networks has attracted lots of attention recently. In this paper, we present an Improved Eigenvector-based Centrality Measures (IECM) for temporal networks by regarding the coupling strength between proximity layers as the inter-layer similarity. Compared with the results of the nodes' influences got by temporal global efficiency for two real networks, the IECM method could identify influential nodes more accurately than the traditional ECM method. Regarding to the fact that different kinds of measurements have different performances, we introduce the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method to measure the global performance. Specially, when the inter-layer coupling strength $\omega$ set as 1 in the ECM method, the accuracy could be averagely enhanced 18.75% and 29.65% at each time layer for Workspace and Enrons datasets respectively, which indicates that measuring the inter-layer coupling strength plays an important role for identifying the influential nodes.

*Key words:* Inter-layer similarity, Temporal networks, Eigenvector centrality, Temporal global efficiency.
*PACS:* 89.75.Da, 05.10.-a

## 1 Introduction

Identifying influential nodes of complex networks has attracted plenty of attention from many branches in scientific community [1,2]. In the last decade, a number of centrality methods have been proposed to identify the influential nodes, such as

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degree centrality [1], betweenness centrality, eigenvector centrality, closeness centrality, and k-shell decomposition [3]. Liu *et al* [4] presented a dynamics-sensitive (DS) centrality by intergrating topological features and dynamical properties. Ren *et al* [5] thought the node importance is not only related to the neighbour size, but also to the clustering coefficient information. Ma *et al* [6] presented a gravity centrality index to identify the influential spreaders in complex networks inspired by the idea of the gravity formula. Bao *et al* [7] presented a heuristic clustering (HC) algorithm on influence maximization for multiple spreaders being distributed dispersively in networks and avoiding the selected nodes to be very "negligible". Lü *et al* [2] presented a systematic comparison of the centrality methods in the existing complex networks.

Nevertheless, these existing methods developed for static networks are restricted to time-independent networks which most real networks belong to. Consequently, it has become a very active research area to extend the centrality methods to temporal networks. Michalski *et al* [8] discussed an interesting way of omitting the time dependence by weighting older paths lighter. Miritello *et al* [9] proposed a way of mapping the dynamic SIR model to a static edge percolation model, where the edge weights of a network whose structure equals that of the aggregated graph are defined in a way that takes into account the temporal correlations and inhomogeneities of edges. Huang *et al* [10] took into account the network dynamics and extend the concept of Dynamic-Sensitive centrality to temporal network, and found that both topological structure and dynamics contribute the impact on the spreading influence of nodes. Wang *et al* [11] proposed a Singular Vector of Tensor (SVT) centrality, which is used to quantitatively evaluate the importance of nodes connected by different types of links in multilayer networks. The method is fast in convergence, efficient and robust in identifying the key nodes. However, it requires a relatively long running time because four quantities need to be calculated.

Taylor *et al* [12] proposed an Eigenvector-based Centrality Measures (ECM) to identify influential nodes for temporal networks. They sliced the network, treated the coupling relationship between the nearest-neighbor time layers as constant parameter $\omega$ that be used to tune interactions between adjacent time layers, and then reshaped the network's associated adjacency tensor into a supra-adjacency matrix. However, how to evaluate the inter-layer weights is crucial for the temporal networks. Inspired by the idea of two sequential snapshots' relationship could be represented by adjacency correlation, we present an Improved Eigenvector-based Centrality Measures (IECM) for temporal networks by considering the coupling strength of neighbour layers as inter-layer similarity, also can be named adjacency correlation. Considering the adjacency correlation calculated by the Pearson correlation, many similarity measurements can also be used to calculate the inter-layer weights, including *Common Neighbour* (CN), *Salton Index* (SAL) [13], *Jaccard Index* (JAC) [14] and so on. Then, an improved supra-adjacency matrix can describe the intra-layer and inter-layer relation for the temporal network. After that, the eigenvector corresponding to the largest singular value of the improved supra-adjacency matrix denotes the centrality of each node $i$ at each time $t$. In order to show the result of our improved method more intuitive, we calculate the Kendall's $\tau$ [15] between our result and the result of the nodes' influence got by the temporal global efficiency for two real networks. The Kendall's $\tau$ can represent the accuracy of our improved IECM method and the traditional ECM method.

Because of the structural property of the temporal network, the performances of one special measurement for different snapshots may be different. Considering this problem, we introduce the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method [16], which reduce the impact of the extremum to seek the ideal alternative, to get the best inter-layer similarity measurement. The results show that our improved method could identify influential nodes more accurately than the traditional ECM method.

## 2 Methods

### *2.1 The traditional Eigenvector-based Centrality Measures*

Normally, when a network $G = (N, E)$ with $N$ nodes and $E$ edges has the time properties, we can name the network as temporal network which can be split into slices. The traditional ECM method [12] described the temporal network $G = (N, E, T)$ with $N$ nodes, $E$ edges and $T$ layers into an $NT * NT$ *supra-adjacency matrix*

$$A = \begin{bmatrix} A^{(1)} & \omega I & 0 & \cdots \\ \omega I & A^{(2)} & \omega I & \ddots \\ 0 & \omega I & A^{(3)} & \ddots \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix}, \tag{1}$$

where $A^{(t)} = \{a_{ij}^t\}$ represents adjacency matrix at time $t$, $a_{ij}^t$ indicates the presence of the edge between node $i$ and node $j$ in time layer $t$, $a_{ij}^t = 1$ if node $i$ is connected with node $j$ in time layer $t$, and $a_{ij}^t = 0$ otherwise. The parameter $\omega$ across all inter-layer edges is known as the inter-layer coupling strength. One can construe $\omega$ to tune interactions between network layers. When $\omega \to 0^+$, the layers become uncoupled; otherwise $\omega \to \infty$, the layers are so strongly coupled that inter-layer weights dominate the intra-layer connections. In this case, the eigenvector corresponding to the largest singular value of the supra-adjacency matrix denotes the centrality of each node $i$ at each time $t$.

### 2.2 The Improved Eigenvector-based Centrality Measures

In this paper, we present an Improved Eigenvector-based Centrality Measures, namely IECM method, to identify influential nodes for temporal networks. Unlike the fixed coupling constant parameter $\omega$ in ECM method, the IECM method argued the layer coupling strength should be calculated by inter-layer similarity. Firstly, we split the network into slices with the rule that the links are classified into a same layer when the connection happens at the same period of time. Secondly, we regard $A^{(t)} = \{a_{ij}^t\}$ as the adjacency matrix at each layer of time $t$, and $C^{(t,t+1)} = \mathrm{diag}(c_1^{(t,t+1)}, c_2^{(t,t+1)}, \ldots, c_N^{(t,t+1)})$ as the inter-layer similarity between the time layer $t$ and $t+1$. $c_j^{(t,t+1)}$ denotes the similarity of node $j$ for two time layers, say $t$ and $t+1$. Thirdly, the network is reshaped into an $NT * NT$ *supra-adjacency matrix* $A$. The eigenvector centrality thinks that the influence of a node is not only determined by the number of its neighbors, but also determined by the influence of each neighbor. So we can get the eigenvector $\nu = \{\nu_1, \nu_2, \ldots, \nu_{NT}\}$, which corresponds to the largest singular value of the *supra-adjacency matrix* $A$, and the $(N(t-1)+i)$-th element of $\nu$ denotes the centrality of the node $i$ at time $t$. In this paper, we consider the condition that the performances of one special measurements for different snapshots may be different for the temporal network. Then the TOPSIS method [16] is introduced to solve this problem, which is developed based on the the idea that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

Figure 1 is an example of a temporal network with four nodes and three time layers. The black solid lines denote the intra-layer edges, and black dotted lines are the inter-layer edges.

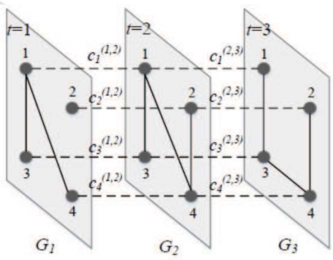


Fig. 1. An example of a temporal network.

In the IECM method, we argue that the node similarity measurements could be introduced to measure the node correlation for the nodes locating in different connected layers. Taking the *Common Neighbour* (CN) similarity measurement and Fig.1 as an example, the details of the IECM method are:

**Step1**: According to the sliced results, let $A^{(t)} = \{a_{ij}^t\}$, where $A^{(t)}$ denotes the adjacency matrix at each layer of time $t$, $a_{ij}^t$ indicates the presence of the edge between node $i$ and node $j$ in time layer $t$, $a_{ij}^t = 1$ if node $i$ is connected with node

$j$ in time layer $t$, and $a_{ij}^t = 0$ otherwise. Then, the adjacency matrix at time layer $1$ can be described as

$$A^{(1)} = \begin{bmatrix} 0\ 0\ 1\ 1 \\ 0\ 0\ 0\ 0 \\ 1\ 0\ 0\ 0 \\ 1\ 0\ 0\ 0 \end{bmatrix}. \tag{2}$$

**Step2**: Let $C^{(t,t+1)} = \mathrm{diag}(c_1^{(t,t+1)}, \ldots, c_N^{(t,t+1)})$ where $C^{(t,t+1)}$ is the inter-layer similarity between the time layer $t$ and $t+1$. $c_j^{(t,t+1)} = \sum_i a_{ij}^t a_{ij}^{t+1}$ denotes the similarity of node $j$ for two time layers, say $t$ and $t+1$. So, the $C^{(1,2)}$ is shown as

$$C^{(1,2)} = \begin{bmatrix} 2\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0 \\ 0\ 0\ 1\ 0 \\ 0\ 0\ 0\ 1 \end{bmatrix}. \tag{3}$$

**Step3**: The improved *supra-adjacency matrix* should be described as

$$A = \begin{bmatrix} A^{(1)} & C^{(1,2)} & 0 \\ C^{(1,2)} & A^{(2)} & C^{(2,3)} \\ 0 & C^{(2,3)} & A^{(3)} \end{bmatrix} = \begin{bmatrix} 0\ 0\ 1\ 1\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0 \\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0 \\ 2\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0 \\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0 \\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1 \\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0 \end{bmatrix}. \tag{4}$$

**Step4**: The eigenvector corresponding to the largest singular value of the improved *supra-adjacency matrix* $A$ denotes the centrality of each node $i$ at each time $t$.

Table 1
Basic statistical features of Workspace and Enrons data sets, $N$ is the number of nodes, $E$ is the number of edges, $C$ is the total interaction times.During denotes the period of time and $T$ is the number of the time layers.

| Data Set | $N$ | $E$ | $C$ | During | $T$ |
|---|---|---|---|---|---|
| Workspace | 92 | 755 | 9827 | 2013.06-2013.07 | 10 |
| Enrons | 151 | 1270 | 33124 | 2001.01-2001.12 | 12 |

There are many similarity measurements for static networks, which can be defined based on the network structure. In this paper, we argue that the node similarity measurements could be introduced to measure the node correlation for the nodes locating in different connected layers. The simplest method is *Common Neighbour* (CN), where the similarity between two nodes are directly given by the number of common neighbours who have connections with them. Considering the degree information of two nodes, variations of the CN index have been proposed, including the *Salton Index* (SAL) [13], *Jaccard Index* (JAC) [14], *Srensen Index* (SOR) [17], *Hub Promoted Index* (HPI) [18], *Hub Depressed Index* (HDI), *Leicht-Holme-Newman Index* (LHN) [19] and *Preferential Attachment Index* (PA) indices. Instead of the number of the common neighbours in the CN index, the *Adamic-Adar Index* (AA) [20], *Resource Allocation Index* (RA) [21], *the Mass Diffusion* (MD) [22], *Heat Conduction* (HC) [23]and *Improved Heat Conduction* (IHC) [24] methods were presented, regarding the node similarity as the summation of their common neighbours' degrees. More detailed introduction is shown in Appendix A.

## 3 Data and Metric

### *3.1 Empirical data sets*

We investigate the performance of the inter-layer similarity-based eigenvector centrality measures for two empirical data sets: Workspace and Enrons. The Workspace [25] is a face-to-face interaction network, where regard employees as nodes and edges set as interactions. The Enrons [26] is an email communication network of U.S. enterprise with potential anomalous email communications spanning over a time range of about 3 years(i.e., from 1999 to 2002). In order to perform a monthly-based analysis, we concentrate on the year 2001 that encompasses the maximum number of emails. The basic statistical properties of data sets are summarized in Table 1.

### 3.2 Evaluation metric

The performance of our method can be measured by the comparison with the nodes' influences. In this paper, we apply the temporal global efficiency to measure the nodes' influences. Following Tang *et al* [27] proposed the temporal global efficiency, which is defined as

$$E = \frac{1}{N(N-1)} \sum_{ij} \frac{1}{d_{ij}}, \tag{5}$$

where $d_{ij}$ is the temporal distance (see Appendix B) from node $i$ to node $j$. High value of $E$ indicates that the nodes of the graphs can communicate efficiently. When deleting node $i$ in time layer $t$, we can get the temporal global efficiency $E_{it}$ at this time. Then, the change between the temporal global efficient $E_{it}$ and the original temporal global efficient $E$ can denote the nodes influence $E'_{it}$, which can be calculated as

$$E'_{it} = |E_{it} - E|. \tag{6}$$

Then, the Kendall correlation coefficient [15] is introduced to measure the accuracy of our improved method. To calculate the kendall's $\tau$, one should provide two sequences data. For each node $i$ at the time layer $t$, we regard $y_{it}$ as its influence and $z_{it}$ as the its centrality, the accuracy of the target centrality in evaluating nodes' influences at each time layer $t$ can be quantified by the Kendall's $\tau$-b [28], as

$$\tau_t = \frac{\sum_{i<j} sgn[(y_{it} - y_{jt})(z_{it} - z_{jt})]}{\sqrt{(n(n-1)/2 - n_1)(n(n-1)/2 - n_2)}}, \tag{7}$$

where $sgn(x)$ is a piecewise function, when $x > 0$, $sgn(x) = +1$; $x < 0$, $sgn(x) = -1$; when $x = 0$, $sgn(x) = 0$. $n$ is the length of the ranking list. $n_1 = \sum_i t_i(t_i - 1)/2$, $n_2 = \sum_j u_j(u_j - 1)/2$, $t_i$ is the number of the $i$-th $y_{it}$ which make $sgn(x) = 0$, $u_j$ is the number of the $j$-th $z_{it}$ which make $sgn(x) = 0$. $\tau$ measures the correlation between two ranking lists, whose value is in the range [-1,1] and the larger $\tau$ to the better performance.

Because of the performances of one special measurement for different snapshots being different, the TOPSIS is introduced. For each inter-layer similarity index $\alpha$, we denote $k_{\alpha j}$ as its Kendall's $\tau$-b at each time layer $j$, creating an evaluation matrix consisting of 38 alternatives and $t$ criteria. Firstly, determine the best alternative and the worst alternative as

$$A_j^* = \max(k_{\alpha j} \mid i = 1, 2, \cdots, m), \tag{8}$$

$$A_j^- = \min(k_{\alpha j} \mid i = 1, 2, \cdots, m). \tag{9}$$

Then, we calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the best condition and the worst is given as

$$D_\alpha^* = \sqrt{\sum_{j=1}^{n}(k_{\alpha j} - A_j^*)^2}, \tag{10}$$

$$D_\alpha^- = \sqrt{\sum_{j=1}^{n}(k_{\alpha j} - A_j^-)^2}. \tag{11}$$

Finally, we rank the alternatives according to the similarity relative closeness to the worst condition

$$C_\alpha = D_\alpha^- / (D_\alpha^* + D_\alpha^-), \tag{12}$$

where $C_\alpha = 1$ if and only if the alternative solution has the best condition; and $C_\alpha = 0$ if and only if the alternative solution has the worst condition.

## 4 Results

We test the performance of IECM method in evaluating the nodes' influences which is quantified by the absolute value of the temporal global efficiency' difference between the node being deleted and not. Two real networks, including a face-to-face interaction network and an email communication network, are used for the empirical analysis. The ECM method is used as benchmark method for comparison. Here we use Kendall's $\tau$-b to measure the correlation between nodes' influences and the considered centrality method, which is in the range [-1,1] and the larger $\tau$ corresponds to the better performance.

Figure 2 shows the accuracy of the IECM method and ECM method at the different time layer $T$. The results show that the Kendall's $\tau$-b of IECM method is almost better than the traditional ECM method, indicating that the ranking lists generated by the IECM method could identify the influential nodes more accurately than the traditional ECM method.

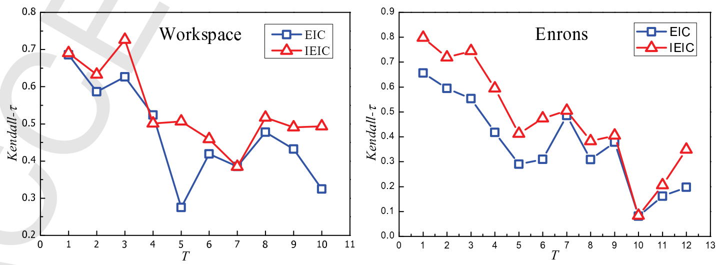


Fig. 2. (Color online)The accuracy of two centrality measures in evaluating nodes' influences in the two real networks, quantified by the Kendall $\tau - b$ when the inter-layer coupling $\omega$ is 1.

The more experiments for different inter-layer coupling strength in the traditional ECM method have been done and the results are shown in Table 2. T is the number

Table 2
The accuracy of different parameter $\omega$ for the Workspace and Enrons data sets. One can find that the accuracy almost get the largest values in IECM method.

| | T | ECM ($\omega$) | | | | | | | | | | | IECM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 1 | 2 | 5 | 10 | 30 | 50 | |
| Workspace | 1 | **0.708** | 0.702 | 0.699 | 0.695 | 0.686 | 0.686 | 0.645 | 0.505 | 0.379 | 0.247 | 0.215 | 0.691 |
| | 2 | 0.601 | 0.601 | 0.596 | 0.593 | 0.588 | 0.586 | 0.579 | 0.550 | 0.473 | 0.366 | 0.327 | **0.632** |
| | 3 | 0.612 | 0.613 | 0.616 | 0.624 | 0.625 | 0.627 | 0.623 | 0.588 | 0.530 | 0.454 | 0.425 | **0.726** |
| | 4 | **0.539** | 0.537 | 0.534 | 0.529 | 0.526 | 0.524 | 0.475 | 0.380 | 0.306 | 0.228 | 0.208 | 0.501 |
| | 5 | 0.258 | 0.259 | 0.260 | 0.265 | 0.270 | 0.275 | 0.275 | 0.215 | 0.171 | 0.130 | 0.123 | **0.507** |
| | 6 | 0.350 | 0.358 | 0.369 | 0.390 | 0.412 | 0.419 | 0.423 | 0.377 | 0.335 | 0.279 | 0.261 | **0.459** |
| | 7 | 0.348 | 0.355 | 0.366 | 0.378 | 0.384 | 0.385 | 0.362 | 0.317 | 0.277 | 0.229 | 0.212 | **0.385** |
| | 8 | 0.469 | 0.469 | 0.469 | 0.469 | 0.475 | 0.477 | 0.485 | 0.430 | 0.392 | 0.329 | 0.310 | **0.517** |
| | 9 | 0.452 | 0.450 | 0.445 | 0.442 | 0.434 | 0.432 | 0.363 | 0.247 | 0.176 | 0.098 | 0.075 | **0.491** |
| | 10 | 0.335 | 0.337 | 0.333 | 0.334 | 0.327 | 0.325 | 0.274 | 0.193 | 0.169 | 0.131 | 0.122 | **0.493** |
| Enrons | 1 | 0.557 | 0.661 | 0.661 | 0.659 | 0.658 | 0.657 | 0.645 | 0.606 | 0.547 | 0.452 | 0.398 | **0.798** |
| | 2 | 0.596 | 0.603 | 0.602 | 0.600 | 0.596 | 0.595 | 0.577 | 0.528 | 0.474 | 0.373 | 0.322 | **0.719** |
| | 3 | 0.557 | 0.557 | 0.556 | 0.554 | 0.554 | 0.553 | 0.542 | 0.516 | 0.493 | 0.436 | 0.407 | **0.745** |
| | 4 | 0.424 | 0.423 | 0.422 | 0.421 | 0.419 | 0.417 | 0.407 | 0.375 | 0.355 | 0.326 | 0.308 | **0.595** |
| | 5 | 0.305 | 0.303 | 0.299 | 0.297 | 0.292 | 0.290 | 0.265 | 0.217 | 0.177 | 0.135 | 0.124 | **0.412** |
| | 6 | 0.313 | 0.313 | 0.311 | 0.310 | 0.310 | 0.310 | 0.305 | 0.280 | 0.258 | 0.232 | 0.222 | **0.475** |
| | 7 | 0.489 | 0.490 | 0.489 | 0.487 | 0.486 | 0.485 | 0.479 | 0.455 | 0.430 | 0.392 | 0.374 | **0.505** |
| | 8 | 0.314 | 0.314 | 0.313 | 0.312 | 0.309 | 0.309 | 0.302 | 0.282 | 0.266 | 0.245 | 0.224 | **0.383** |
| | 9 | 0.381 | 0.380 | 0.379 | 0.378 | 0.379 | 0.378 | 0.371 | 0.360 | 0.329 | 0.282 | 0.267 | **0.405** |
| | 10 | 0.078 | 0.078 | 0.079 | 0.079 | 0.081 | 0.081 | 0.091 | 0.109 | **0.120** | 0.109 | 0.097 | 0.083 |
| | 11 | 0.165 | 0.164 | 0.163 | 0.162 | 0.161 | 0.162 | 0.154 | 0.138 | 0.123 | 0.082 | 0.061 | **0.205** |
| | 12 | 0.200 | 0.200 | 0.199 | 0.197 | 0.198 | 0.198 | 0.188 | 0.170 | 0.159 | 0.128 | 0.115 | **0.349** |

of the time layer

Finally,as shown in Fig.3, the improved ratio $\eta$ of Kendalls $\tau$-b generated by the IECM method comparing with the results of the ECM method is almost larger than 0, indicating that the IECM method performs better than the ECM method. The improved ratio $\eta$ is defined as

$$\eta = \frac{\tau^{new} - \tau^0}{\tau^0}, \tag{13}$$

where $\tau^{new}$ is Kendalls $\tau$-b of the IECM method, and $\tau^0$ is Kendalls $\tau$-b obtained by the ECM method. Clearly, $\eta > 0$ indicates an advantage of IECM method. In the Workspace network, the largest improved ratio $\eta$ generated by $\omega = 1$ could reach 84.3% in time layer 5. In the Enrons network, the largest improved ratio $\eta$ generated by $\omega = 1$ could reach 76.27% in time layer 12. Furthermore, we calculated the mean of the improved ratio $\eta$. In the Workspace network, Kendalls $\tau$-b could be enhanced 18.75%. In the Enrons network, Kendalls $\tau$-b could be enhanced 29.65%.

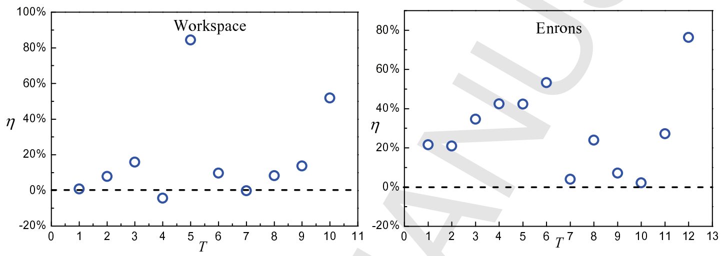


Fig. 3. (Color online)The improved ratio $\eta$ of different time layers $T$ in the Workspace and Enrons networks.

## 5 Conclusion and discussions

Taking into account the inter-layer similarity of the "identity edges", we propose an Improved Eigenvector-based Centrality Measure (IECM) for temporal network to identify the influential nodes. First, we slice the temporal network according to time interval. Then, the supra-adjacency matrix could be used to describe the collection of both the temporal network edges (which are intra-layer edges) and the "identity edges" (which are inter-layer edges) that couple the node-layer pairs $(i, t)$ for the same physical node $i$ across the $T$ network layers by inter-layer similarity. With that, the eigenvectors corresponding to the largest singular value of the supra-adjacency matrix denotes the centrality of each node $i$ at each time $t$. Finally, we calculate the kendalls $\tau$ value between the nodes influence got by temporal global efficiency and the nodes centrality got by supra-adjacency matrix for evaluating the accuracy of our method. Meanwhile, the TOPSIS method is introduced to determine the ideal inter-layer similarity measurement. The experimental results for the empirical networks show that the performance of IECM method can improve larger than the traditional ECM method.

However, in this paper, the coupling relationship between layers is only limited to the consideration of two sequential snapshots. In real life, the connection between the nodes is not only dependent on the information about neighbour layers, but also relies on the multiple segment information in time. Moreover, the research of this paper only applies to small-scale data, and how to study the case of big data should be discussed later. And the selection of data sets will have an impact on the experimental results. In addition, temporal networks have attracted more and more attention recently. How to design a new iterative resource allocation method in these networks would be investigated in the future.

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**Appendix A: Inter-layer Similarity Measurements**

We use $k_i^t = a_{ij}^t a_{ij}^{t+1} \sum_j a_{ij}^t$ and $k_i^{t+1} = a_{ij}^t a_{ij}^{t+1} \sum_j a_{ij}^{t+1}$ to represent the selection times of node $i$ in time layer $t$ and $t+1$ respectively. Then, the popularity $k_i$, the number of nodes that node $i$ have selected between two sequential snapshots, can be got by calculating the minimum(min), maximum(max), summation(sum), average(mean), multiplied(mul) and the number of the common neighbours between two sequential snapshots (scn) between $k_i^t$ and $k_i^{t+1}$

$$k_i = \min(k_i^t, k_i^{t+1}), \tag{14}$$

$$k_i = \max(k_i^t, k_i^{t+1}), \tag{15}$$

$$k_i = (k_i^t + k_i^{t+1})/2, \tag{16}$$

$$k_i = k_i^t k_i^{t+1}, \tag{17}$$

$$k_i = k_i^t + k_i^{t+1}, \tag{18}$$

$$k_i = k_i^t + k_i^{t+1} - \sum_j a_{ij}^t a_{ij}^{t+1}. \tag{19}$$

With these defined parameters, the inter-layer similarity measurements referred in this paper read:

$$CN : c_j^{(t,t+1)} = \sum_i a_{ij}^t a_{ij}^{t+1}, \tag{20}$$

$$SAL : c_j^{(t,t+1)} = \frac{\sum_i a_{ij}^t a_{ij}^{t+1}}{\sqrt{[\sum_i a_{ij}^t][\sum_i a_{ij}^{t+1}]}}, \tag{21}$$

$$JAC : c_j^{(t,t+1)} = \frac{\sum_i a_{ij}^t a_{ij}^{t+1}}{\sum_i a_{ij}^t + \sum_i a_{ij}^{t+1} - \sum_i a_{ij}^t a_{ij}^{t+1}}, \tag{22}$$

$$SOR : c_j^{(t,t+1)} = \frac{2\sum_i a_{ij}^t a_{ij}^{t+1}}{\sum_i a_{ij}^t + \sum_i a_{ij}^{t+1}}, \tag{23}$$

$$HPI : c_j^{(t,t+1)} = \frac{2\sum_i a_{ij}^t a_{ij}^{t+1}}{\min[\sum_i a_{ij}^t, \sum_i a_{ij}^{t+1}]}, \tag{24}$$

$$HDI : c_j^{(t,t+1)} = \frac{2\sum_i a_{ij}^t a_{ij}^{t+1}}{\max[\sum_i a_{ij}^t, \sum_i a_{ij}^{t+1}]}, \tag{25}$$

$$LHN : c_j^{(t,t+1)} = \frac{\sum_i a_{ij}^t a_{ij}^{t+1}}{\sum_i a_{ij}^t \sum_i a_{ij}^{t+1}}, \tag{26}$$

$$PA : c_j^{(t,t+1)} = \sum_i a_{ij}^t \sum_i a_{ij}^{t+1}, \tag{27}$$

$$AA : c_j^{(t,t+1)} = \sum_i \frac{1}{\log k_i}, \tag{28}$$

$$RA : c_j^{(t,t+1)} = \sum_i \frac{1}{k_i}, \tag{29}$$

$$MD : c_j^{(t,t+1)} = \frac{1}{k_i^t} \sum_i \frac{1}{k_i}, \tag{30}$$

$$HC : c_j^{(t,t+1)} = \frac{1}{k_i^{t+1}} \sum_i \frac{1}{k_i}, \tag{31}$$

$$IHC : c_j^{(t,t+1)} = \frac{1}{(k_i^{t+1}} \sum_i \frac{1}{k_i)^2}. \tag{32}$$

## Appendix B: Definition of temporal distance

The concept of temporal distance, which is introduced detailed and clear in Ref. [27], means the fastest possible way from one node to the other nodes. As shown in Fig.1, we assume that node 1 can start passing the message at time $t = 1$, and the message has to be delivered by time $t = 3$. On graph $G1$, node 1 can directly pass the message to nodes 3 and 4. Then, the temporal distance from 1 nodes 3 or 4 will be set as 1 for their link in one unit of time. Due to the message from node 1 to 2 should be passed from 1 to 4 in graph G1, and then from node 4 to node 2 in G2, with their link in two time units, the temporal distance is 2. Temporal distance of nodes in temporal network of Fig.1 is presented in Table 3. Notice also that, even if the time-varying graph consists of a sequence of undirected graphs, the temporal distances are not symmetric for the time order of the links.

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Table 3
Temporal distance of nodes in temporal network of Fig.1.

| Node | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 1 |
| 2 | $\infty$ | 0 | 3 | 2 |
| 3 | 1 | 3 | 0 | 2 |
| 4 | 1 | 2 | 2 | 0 |


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