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Opinion formation and bi-polarization with biased assimilation and homophily

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HIGHLIGHTS

- An agent-based model considering biased assimilation and homophily is proposed.
- Biased assimilation can contribute to opinion bi-polarization and accelerate convergence.
- Biased assimilation and homophily acting in tandem can largely lead to opinion bi-polarization.
- Homophily is necessary for the population to form opinion bi-polarization on complete network.
- Larger rewiring probability p and average degree of each node $\langle K \rangle$ of small world network can contribute to opinion consensus and accelerate convergence.

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ABSTRACT

An agent-based model incorporating biased assimilation is proposed in this paper to investigate opinion dynamics over a connected social network. The opinion of each agent is represented by a sequence of arguments, and it evolves through the interactions between agents. The probability that one agent chooses another to communicate depends on the similarity of their opinions. During every interaction, interacting agents exchange the argument randomly selected from the corresponding arguments sequence. Theoretical analysis reveals that this model results in consensus on either extreme positive opinion or extreme negative opinion, or generates bi-polarization. Numerical simulations are carried out to investigate the dynamics of the model over different networks. Results are obtained in terms of the effect of homophily, biased assimilation and network topology on opinion formation.

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1. Introduction

Research of opinion dynamics on given issues is rapidly emerging as an important inter-disciplinary topic in last decades. Up to now, lots of works have been presented to explore this area by drawing support from social influence theory, agent-based modeling methodology and network science [1,2]. For example, Refs. [3–7] studied opinion dynamics by postulating that individuals suffer from group pressure so that they adjust themselves toward the opinion that lies within their acceptance levels. In these works, each agent updates its opinion to a convex combination of its own opinion and the interactive agent's, i.e. opinion update process is implemented by means of averaging mechanism (also called compromise), which will eventually decrease the opinion difference. Although in views of social influence this mechanism appears to be

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realistic when agents with different opinion come to interact with each other, it implies interactive agents keep their opinion unchanged when they have already agree with each other. It is due to this feature that extreme opinion or polarization cannot be observed as a consequence of the models based on averaging mechanism. Thus other models, for example Refs. [8–12], are proposed based on persuasive argument exchange mechanism, where agents communicate with each other by exchanging their arguments rather than opinion value represented by a real number. Such mechanism is less explored compared to the averaging mechanism. One advantage over the models based on averaging mechanism lies in that when agents happen to interact with those holding the same opinion as theirs, they can intensify their previous opinion, leading to more extreme in their stance, such that agents with extreme opinion can be observed as a consequence of the model.

Polarization, on the other hand, is widely observed in the real world, such as political polarization [13], cultural polarization [14], and so on. However, there is not yet a unanimous agreement on how to explain polarization [15]. Various models have been presented as well from different perspective to mimic the process of the polarization. Ref. [16] proposed a model which can result in two opposing extreme opinions, by considering both positive and negative influence from the interactive agents. Ref. [11] developed a model of argument-communication theory of bi-polarization (ACTB), which can give rise to bi-polarization without negative influence, if the strength of homophily is large enough. Ref. [10] investigated the influence of homophily and demographic crisscrossing on opinion dynamics and found in teams with strong faultlines opinion polarization occurs in the short run but consensus will eventually be reached in the long run. Ref. [12] presented a model exploring the influence of compromise and persuasive-argument mechanism, and found that if the strength of persuasion outweighs that of compromise, polarized opinion arises. Ref. [17] proposed a modified DeGroot model incorporating the effect of biased assimilation to analyze the opinion dynamics and showed that if the agents are sufficiently biased, the opinion process would result in polarization.

In this paper, we propose a model based on persuasive argument theory and biased assimilation to analyze the dynamics of opinion formation process. Firstly, inspired by Refs. [10–12], we adopt the mechanism of argument exchange, where agents' opinions on the given issue are based on pro and con arguments. Secondly, inspired by Ref. [18], which showed that individuals are likely to process relevant information in a biased manner, whereby they are apt to accept confirming evidence and critically evaluate disconfirming evidence, we thus incorporate this influence in the model. It is worth pointing out that to the best of our knowledge, there are few works [17] on such field. The proposed model may provide further insight to investigate the dynamics of opinion bi-polarization. Inspired by Refs. [10,11], which indicates that opinion polarization occurs rarely with moderate homophily, this paper will study the influence of strength of homophily on opinion polarization as well. Moreover, most of Refs. [10,11,17] available are assumed the underlying network is complete network, thus the influence of different network topologies on the opinion bi-polarization will be explored as well in this paper.

The rest of this paper is organized as follows. The model formulation is introduced in next section, followed by brief theoretical analysis of the model. The main results are presented in Section 4, devoted to the analysis of influence of biased assimilation, homophily and social network topology on the dynamics of opinion bi-polarization by numerical simulations carried on different networks. Finally, some conclusions are given in Section 5.

2. Model

We consider a society composed of n social agents denoted by $V = \{1, 2, 3, \dots, n\}$, whose opinion formation process unfolds over a social network, represented by a connected undirected graph $G = (V, E)$, where $E \subset V \times V$ is the set of edges. Denote by N_i the set of neighbors of agent i, i.e. $N_i = \{j \in V | (i, j) \in E\}$. For every agent $i \in V$, denote by x_i its opinion representing its stance on a given issue.

Based on persuasive argument exchange mechanism and inspired by Ref. [11], each agent's opinion x_i on a given issue is represented by a finite number of pro and con arguments, where pro argument can mean to be supportive of the given issue while con argument can mean to be against the given issue. In this paper, we assume all of the agents have L arguments to represent their opinion. The valence of the argument of x_i, denoted by $a_{i,k}$ $(k = 1, 2, \dots, L)$, is determined by $+1$ for pro argument and -1 for con argument. The agent i's opinion value at time t, $x_i(t)$, is the average value of its arguments, i.e. $x_i(t)$ can be expressed by Eq. (1). Obviously, $x_i(t) \in [-1, 1]$, and $x_i(t) = 1$ when all of the arguments of agent i are pro, while $x_i(t) = -1$ when all of the arguments are con. The extreme value -1 $(+1)$ corresponds to the extremist which is the most strongly against (in favor of) the given issue.

$$x_i(t) = \frac{1}{L}\sum_{k=1}^{L} a_{i,k}. \tag{1}$$

The opinion formation process is a sequence of interactions between agents. Each interaction entails two phases: (i) agent i selects one agent j from its neighbors to interact with, (ii) agent i communicates with agent j by exchanging argument and assimilate agent j's argument in a biased manner. In what follows, we will describe how to choose the interactive partner and how to model the mechanism of biased assimilation.

2.1. Choose the interactive partner

For every agent $i \in V$, it will pick up one of its neighbors to communicate with at every time period. The probability by which agent $j \in N_i$ is chosen by i at time t, $p_{ij}(t)$, depends on how close its opinion is to agent i's at previous time $t - 1$.

It is reasonable to assume that agents with similar opinion are more likely to interact with each other. Define the degree to which how close the opinion between i's and j's at time $t-1$ as $od_{ij}(t-1) = \mathrm{e}^{-|x_i(t-1)-x_j(t-1)|}$. This form is commonly used in economics and physics [19]. Obviously, $0 < od_{ij} \le 1$, and od_{ij} reaches its maximum 1 only when agents i and j hold the same opinion. Then the probability $p_{ij}(t)$ can be defined by Eq. (2)

$$p_{ij}(t) = \frac{\mathrm{e}^{-\beta|x_i(t-1)-x_j(t-1)|}}{\sum_{k\in N_i} \mathrm{e}^{-\beta|x_i(t-1)-x_k(t-1)|}} \tag{2}$$

where $\beta \ge 0$ is a constant and can be referred to the measure of openness degree of the society. If β is large, the agents are conservative and inclined to only communicate with those agents whose opinions are similar to them. Contrarily, when β is small, agents are ready to interact with those who are different from them in opinions. In this sense β can be regarded as the strength of homophily, and this is similar to the parameter h presented in Ref. [11]. In this paper, Homophily, just the same as Refs. [11,17,20], means greater interaction between similar agents. It is worth to note that this is based on the fact that the underlying network structure is complete network and any two individuals can communicate with each other. In this sense, if the underlying network is not complete network, for instance small-world network or scale-free network, it can be regarded as the homophilous network. Note that when $\beta = 0$, agent i would select its interactive partner from its current neighbor set uniformly. Thus for the complete network, i.e. $N_i = \{j \in V | j \neq i\}$, $\beta = 0$ also means there is not the effect of homophily.

The parameter β can be agent-specific, but in order to simplify the analysis in this paper it is assumed to be the same for all agents. The effect of β on the opinion dynamics will be analyzed in detail later.

2.2. Biased assimilation

Biased assimilation is a well-known phenomenon in social psychology [18] and indeed leads to more extreme opinions. Biased assimilation can be described intuitively by the behavior that agents readily accept the confirming evidence at face value and critically examine the disconfirming evidence. Inspiring the result in Ref. [17], which characterized the biased manner by means of Urn process, we model the effect of biased assimilation on opinion formation as follows: (1) during each interaction, the interactive agent j chooses one argument $a_{j,l}$ $(0 < l \le L)$ randomly out of its L arguments, (2) agent i selects one argument $a_{i,l'}$ uniformly at random from its own L arguments and compare it with $a_{j,l}$. If $a_{i,l'} = a_{j,l}$, then agent i adopts $a_{j,l}$, i.e. adding the same argument as a_{jl} to its argument set, and discards one arguments chosen randomly from its argument set; otherwise, agent i will reject agent j's argument and keeps its own argument set unchanged. It is worth to note that if agent i simply accepts the argument $a_{j,l}$ and discards one of its own argument chosen randomly, then it is said that agent i deals with the interactive information without biased assimilation. It can be indicated obviously that there is one case where all of the agents will always remain their arguments set unchanged: one is that all of the argument of agent i are pro (con), while those of agent j are con (pro). The effect of biased assimilation will be analyzed later as well.

3. Dynamics

We study the dynamics of proposed model by analyzing the evolution of the number of pro and con arguments of each agent. For every agent $i \in V$, denote by $n_i^+(t)$ the number of pro arguments of agent i at time step t, and $n_i^-(t)$ the number of con arguments of agent i, where $n_i^+(t) + n_i^-(t) = L$. As is indicated by the model described before, for every agent $i \in V$ at time step t, when it is interacting with agent j, its number of pro arguments n_i^+ will add by 1 with the probability $\frac{n_j{}^+(t)}{L}\frac{n_i{}^+(t)}{L}\frac{n_i{}^-(t)}{L+1}$, and reduce by 1 with the probability $\frac{n_j{}^-(t)}{L}\frac{n_i{}^-(t)}{L}\frac{n_i{}^+(t)}{L+1}$. Thus, by following mean-field equation, the number of pro arguments of agent i, $n_i^+(t)$, evolves according to Eq. (3). We can obtain the equation for the number of con arguments $n_i^-(t)$ easily by the transformation $n_i^+ \leftrightarrow n_i^-$ and $n_j^+ \leftrightarrow n_j^-$ in Eq. (3).

$$\frac{\mathrm{d}n_i^+(t)}{\mathrm{d}t} = \sum_{j\in N_i(t)} p_{ij}\left[\frac{n_j{}^+(t)}{L}\frac{n_i{}^+(t)}{L}\frac{n_i{}^-(t)}{L+1} - \frac{n_j{}^-(t)}{L}\frac{n_i{}^-(t)}{L}\frac{n_i{}^+(t)}{L+1}\right]. \tag{3}$$

The first term of the right side of Eq. (3) accounts for the gain of pro arguments, i.e. agent i adopts one pro argument and discards one con argument, while the second term describes the loss of pro arguments, i.e. agent i adopts one con argument and discards one pro argument. By factorization, Eq. (3) can be reducible to Eq. (4).

$$\frac{\mathrm{d}n_i^+(t)}{\mathrm{d}t} = \sum_{j\in N_i(t)} p_{ij}\frac{n_i{}^+(t)[L-n_i{}^+(t)][n_j{}^+(t)-n_j{}^-(t)]}{L^2(L+1)}. \tag{4}$$

By setting the left side of Eq. (4) $\frac{\mathrm{d}n_i^+(t)}{\mathrm{d}t} = 0$, Eq. (4) can be solved by $n_i^+(t) = 0$, $n_i^+(t) = L$, and $n_j^+(t) = n_j^-(t)$. However, $n_j^+(t) = n_j^-(t)$ is not a stable solution for the proposed model, since it only indicates that during the interaction with agent

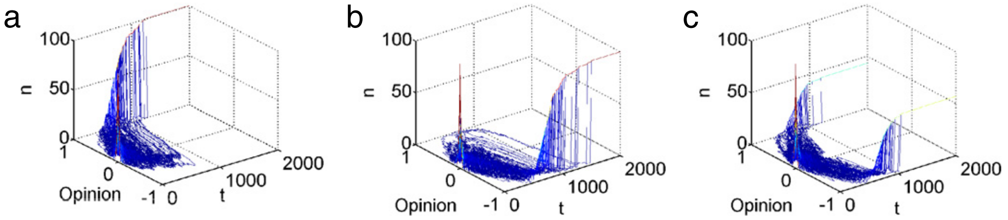


Fig. 1. The three possible types of opinion formation of the proposed model, where $n = 100$, $L = 60$ and each agent is assumed to have the same number of pro arguments and con arguments for its initial opinion, i.e. $x_i(0) = 0$. (a) Consensus in extreme opinion $+1$, (b) consensus in extreme opinion -1, (c) bi-polarization.

j, agent i adopts one pro argument or one con argument with the same probability. The two trivial solutions $n_i^+(t) = 0$ and $n_i^+(t) = L$, which correspond to the two extremes $x_i(t) = -1$ and $x_i(t) = 1$, respectively, are stable, since according to the mechanism of biased assimilation, it can be drawn obviously that agent i will always keep its argument set unchanged. Obviously, the proposed model surely results in either complete consensus or maximal bi-polarization. Consensus is reached when the opinions of all of the agents converge to the extreme value, $+1$ or -1; bi-polarization is realized if some of the agents converge to $+1(-1)$, while the rest converge to the opposing extreme opinion. The three possible equilibriums are shown in Fig. 1, where the number of agents $n = 100$, $L = 60$, $\beta = 2$ and all of the agents have the same number of pro and con arguments as their initial opinion, i.e. $n_i^+(0) = n_i^-(0) = 30$ and $x_i(0) = 0$.

4. Numerical simulations and analysis

Numerical simulations are carried out in this section to analyze the influence of homophily, biased assimilation and network topology on the dynamics of the proposed model.

We replicated the simulations by considering initial opinion with both homogeneous and non-homogeneous distribution in the opinion scale, and found higher probability to reach bi-polarization with non-homogeneous initial opinion. Likewise, we replicated the simulations with different value of L from 10 to 100 and found it is less likely to result in bi-polarization with larger L, vice versa, which has also been shown in Ref. [11]. This can be explained by the fact that it is more difficult for the group to result in bi-polarization when there are more arguments on the given issue for them to consider. In this experiment when $L > 50$, the proportion of bi-polarization keeps almost unchanged. In the following simulations results, we assume $L = 60$ for each agent, and all of the agents start with identical opinion at the middle of opinion scale, i.e. $x_i(0) = 0$ for all $i \in V$, by assigning to each agent a random set of arguments with the same number of pro and con arguments.

In order to investigate the effect of biased assimilation better, we also define the degree of biased assimilation by p_b, where $p_b \in [0, 1]$. When $p_b = 0$, agent accepts the interactive information without biased assimilation, while when $p_b = 1$, agent always assimilates the interactive information with bias. Simulations are carried out in Karate network [21], complete network, and small world network [22] respectively.

4.1. Results on Karate network

Karate network, also known as Zachary's Karate Club, describes friendships between 34 members of a karate club at a US university in the 1970. The data was first presented by Zachary in Ref. [21] to study problem of fission in small groups. The topology of the network is shown in Fig. 2, where each node represents a member of the club and each edge represents a tie between two members of the club. The simulation is run dependently for 100 times under the same condition as well.

Fig. 3 shows the influence of the strength of homophily β and biased assimilation p_b on group bi-polarization. It can be obviously observed that either larger β or larger p_b can contribute to resulting in opinion bi-polarization. For the fixed β (p_b), the proportion of the runs ending in opinion bi-polarization increases as p_b (β) increases. When homophily and biased assimilation act in tandem, opinion bi-polarization can occur more easily, especially when the value of β is moderate, for example $\beta = 5$, moderate degree of biased assimilation can largely contribute to bi-polarization. However, when the strength of homophily β is rather large, for example $\beta = 12$, the effect of biased assimilation is minor. Likewise, when p_b is large enough, the effect of homophily is also subtle. It can be also seen that when $\beta = 0$, even if p_b is quite large, there is only small proportion of runs ending in bi-polarization, while when $p_b = 0$, as long as β is large enough, the proportion of runs resulting in bi-polarization would be relatively large. This indicates that homophily plays more important role on opinion bi-polarization.

In order to further look into the influence of β and p_b on opinion bi-polarization, we also evaluate for each run the degree to which the population is bi-polarization at the equilibrium point. We define the *degree of bi-polarization* as the distance from the maximal bi-polarization, which is the case when the whole population evolve into two equally large subgroups holding opposing opinions. Denote by γ the degree of bi-polarization for each simulation run, then γ can be given by Eq. (5).

$$\gamma = 1 - \frac{|n^{+1} - n^{-1}|}{n} \tag{5}$$

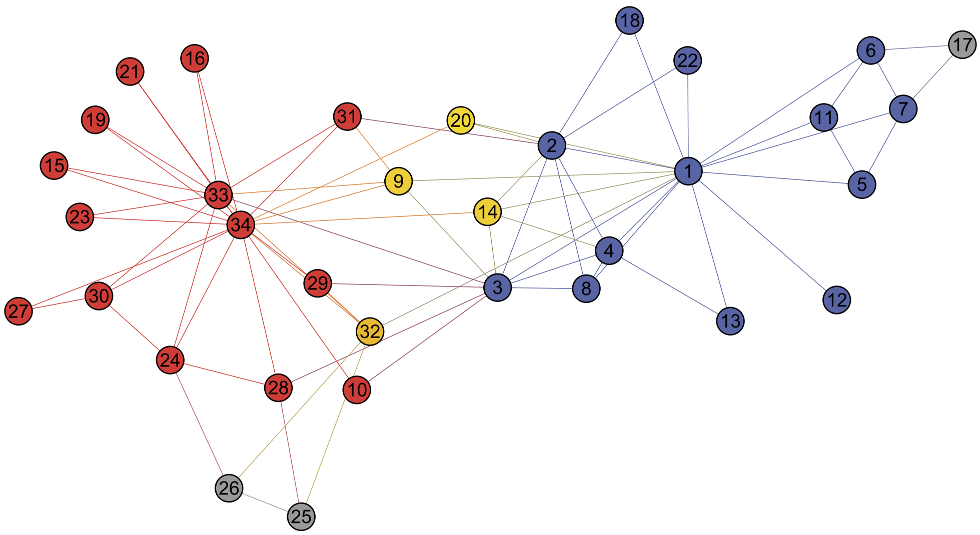


Fig. 2. Topology of Karate network.

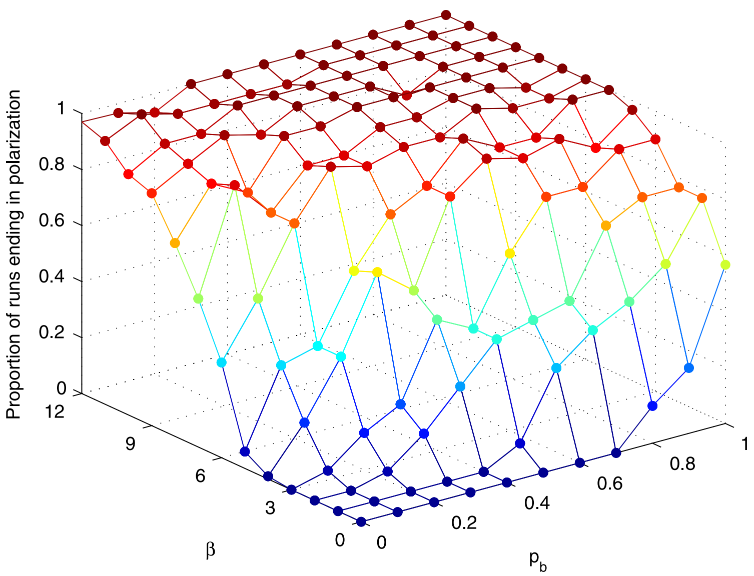
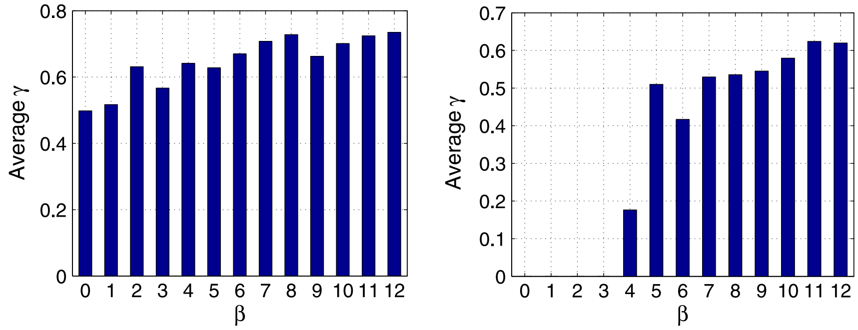


Fig. 3. The proportion of the independent runs that end in bi-polarization with different strength of homophily β and different strength of biased assimilation p_b.

where n^{+1} is the number of agents with opinion at $+1$, and n^{-1} as the number at -1 in each simulation run. Recalling the analysis in the last section, we know the opinion of each agent results in either $+1$ or -1, thus $n^{+1} + n^{-1} = n$. Obviously $\gamma \in [0, 1]$, and when $n^{+1} = n$ or $n^{-1} = n$, i.e. consensus is achieved, $\gamma = 0$, while when $n^{+1} = n^{-1}$, i.e. maximal bi-polarization, $\gamma = 1$. If the simulation run ends in bi-polarization, $\gamma \in (0, 1]$. Here we are only interested in bi-polarization, thus the case where $\gamma = 0$ is not considered, i.e. only referring to the runs that end in bi-polarization.

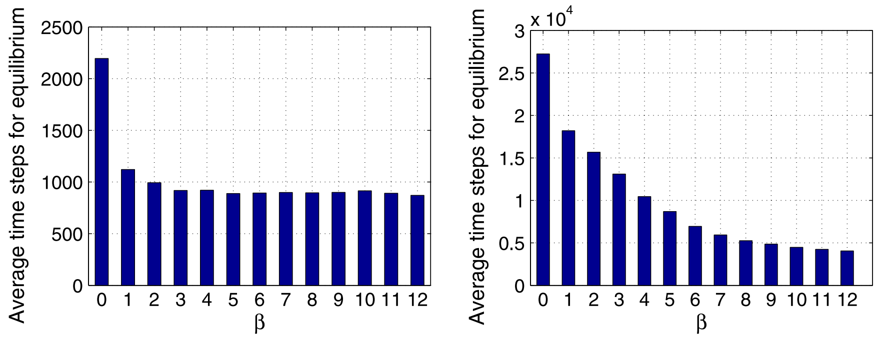
Fig. 4 shows the average degree of bi-polarization of the realizations that end in bi-polarization. Here for brevity only the results when $p_b = 0$ and $p_b = 1$ are presented. It can be observed from panel (a) that the average γ increases smoothly with larger β, but the difference is small between each other. In panel (b) the average degree of bi-polarization also increases slightly as β increases. Since all of the runs result in consensus for $\beta \leq 3$ when $p_b = 0$, the average γ in panel (b) is zero below $\beta = 3$. It can be also observed that the average γ in panel (a) is larger than the corresponding point in panel (b), which indicates that the mechanism of biased assimilation can enhance the degree of bi-polarization.

We are also interested in the influence of the strength of homophily β and the strength of biased assimilation p_b on convergence speed. To this end, the average time steps for the runs taking to reach equilibrium as a function of β and p_b is



(a) $p_b = 1$. (b) $p_b = 0$.

Fig. 4. Average degree of bi-polarization with different β and p_b. The results refer to the runs which result in bi-polarization.



(a) $p_b = 0$. (b) $p_b = 1$.

Fig. 5. Average time steps for equilibrium with different β and p_b.

shown in Fig. 5, where also only the results when $p_b = 0$ and $p_b = 1$ are presented. It can be seen from panel (a) that the population takes almost the same average steps to reach the stable equilibrium with different β except the original point at $\beta = 0$, where the population takes much more steps. The result is very similar to Fig. 11. By and large, larger β cannot accelerate convergent speed for the model when $p_b = 1$. However, in panel (b) it can be observed the average convergence time varies inversely with β. Moreover, it can be seen by comparing panel (a) with (b) that the average convergence time with $p_b = 1$ is much less than that with $p_b = 0$. This indicates that the mechanism of biased assimilation can accelerate convergence.

4.2. Results on complete network

In this experiment, the number of agents is set as $n = 100$. The simulation under the same condition is realized for 100 times independently. The degree of biased assimilation is assumed as $p_b = 1$. Fig. 6 shows the proportion of runs that ends in bi-polarization as a function of β. It indicates that the agents are more likely to result in bi-polarization with larger β. In this experiment, when $\beta \geq 3$, the population almost surely ends in bi-polarization, but when $\beta = 0$, i.e. without the effect of homophily, the proportion is very small and there are only two out of the 100 realizations ending in opinion bi-polarization.

The average degree of bi-polarization with different β for the runs ending in bi-polarization is shown in Fig. 7. It can be seen that the average γ increases with larger β. There is a threshold for the influence of β. When $\beta > 9$ in this experiment, the average γ reaches its plateau at nearly 0.9, which indicates that the two polarized subgroups are almost well-matched in the number. In addition, when $\beta = 0$, the degree of bi-polarization is very weak, near to 0. Actually, we also replicate the simulations with different n, and find that in some cases where n is small, none of runs end in bi-polarization at $\beta = 0$. This may be because if the number of people is smaller, it is more easily for them to reach consensus. Combining with the result in Fig. 6, it can be drawn that bi-polarization is very reluctantly reached without homophily.

Fig. 8 presents the average convergence time with different β. It can be observed that convergence time decreases slightly with larger β compared to small β, but the difference is very indistinct. This may be due to the fact that convergence speed largely depends on the connectivity of the underlying network and complete network has large algebraic connectivity. In this case, the effect of homophily is diminished, thus the convergence time differs slightly with different β. Thus it is hard to say that larger β can accelerate convergence in this experiment.

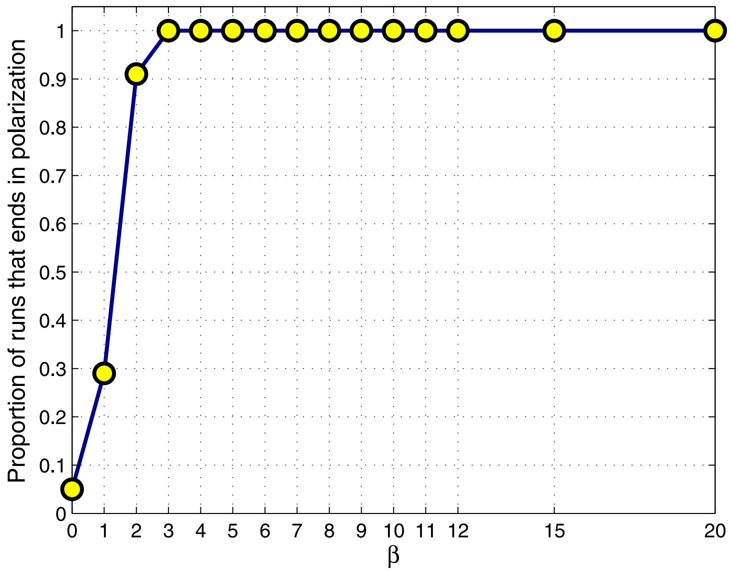


Fig. 6. Average proportion of runs that end in opinion bi-polarization as a function of β.

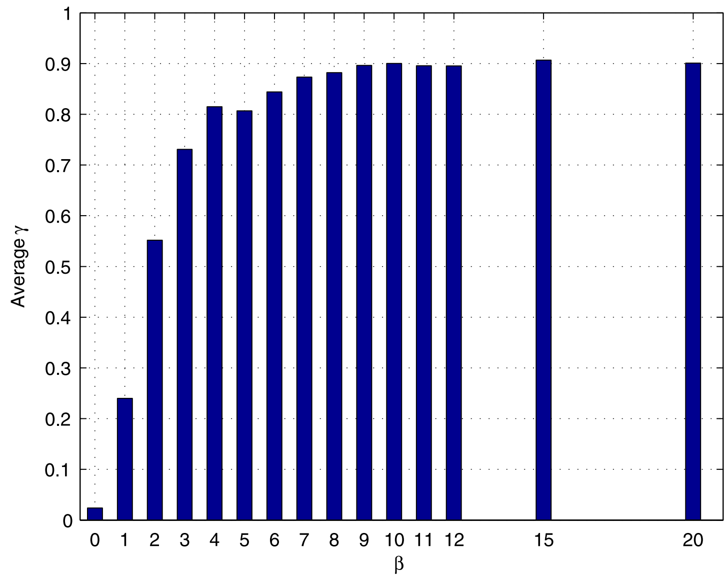


Fig. 7. Average degree of bi-polarization of the runs that end in bi-polarization with different β.

4.3. Results on small world network

In this experiment, the population size is also assumed as 100, the degree of biased assimilation is assumed as $p_b = 1$ and the small world network is generated by WS model proposed in Ref. [22], where the initial degree of each node $K = 12$ and the rewiring probability of each edge $p = 0.2$.

Fig. 9 shows the proportion of runs that end in opinion bi-polarization with different β. It can be observed that the proportion increases as β becomes larger and in this experiment when $\beta \geq 2$, the simulation results in opinion bi-polarization surely. However, in the WS network when $\beta = 0$ the proportion of runs ending in bi-polarization is much large, which is very different from Fig. 6. This is because as mentioned in Section 2.2 WS network is implicitly homophilous in structure, where each agent cannot visit all other agents with the same probability even when $\beta = 0$, because each agent does not connect to all other agents in S–W network.

Similar to the previous subsection, we also give the influence of β on the average degree of bi-polarization γ in Fig. 10. It can be seen that the average γ increases as β increases and then reaches its plateau when β is large enough. This indicates that homophily indeed can enhance the degree of bi-polarization. In addition, when $\beta = 0$, the corresponding $\overline{\gamma}$ is much

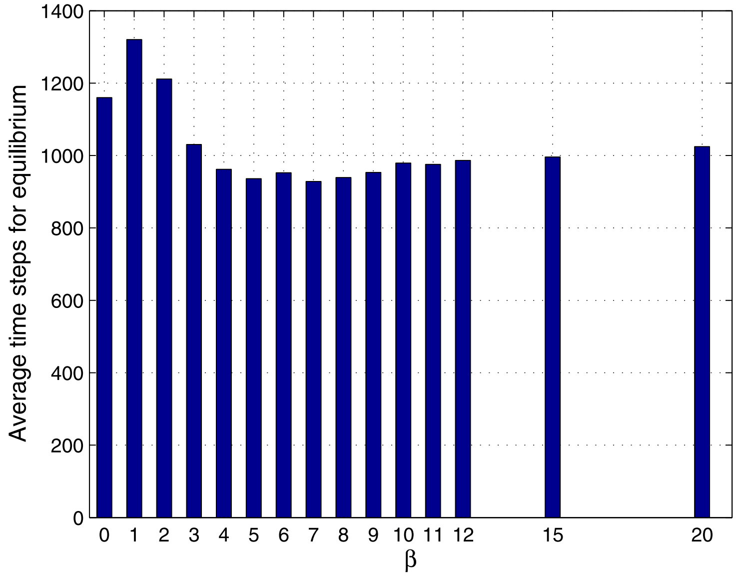


Fig. 8. Average time steps for all of the runs to reach equilibrium with different β.

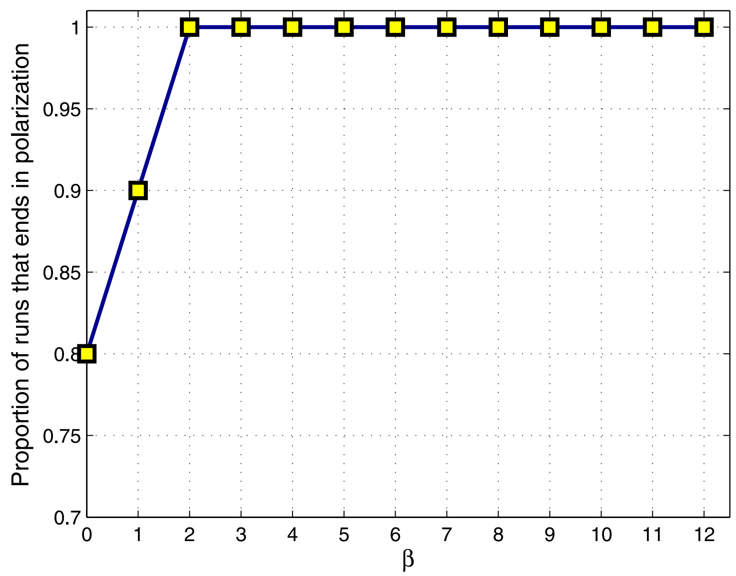


Fig. 9. The proportion of runs that end in opinion bi-polarization with different β.

larger than 0, which indicates furthermore that the final opinion of the population can result in bi-polarization in the small network without homophily.

Fig. 11 presents the average convergence time steps with different β. It indicates although the presence of homophily can speed up convergence compared to population without homophily, larger strength of homophily has little impact on convergent speed in this experiment.

In order to investigate the influence of network topology further, we also study the influence of the rewiring probability p and the average degree $\langle K \rangle$ on opinion dynamics.

Firstly, the result of the influence of p on the opinion dynamics is shown in Figs. 12–14, where $n = 100$, $\langle K \rangle = 12$ and $\beta = 0$. The result refers to 100 independent realizations. Fig. 12 is the proportion of the runs that ending in bi-polarization with different p. It can be seen that the proportion of the runs ending in bi-polarization declines as p increases. This indicates that larger p can prevent the population from resulting in bi-polarization. Moreover, it can be observed from Fig. 12 that if p is relatively small (in this experiment $p < 0.1$), the proportion declines slightly, while if p is relatively large ($0.1 < p < 0.2$), the proportion decreases sharply. This may be explained by the fact that when p is very small, the network is close to regular network, where the agents are relatively isolated if $\langle K \rangle$ is much less than n. As a result, the agents are more likely to result in

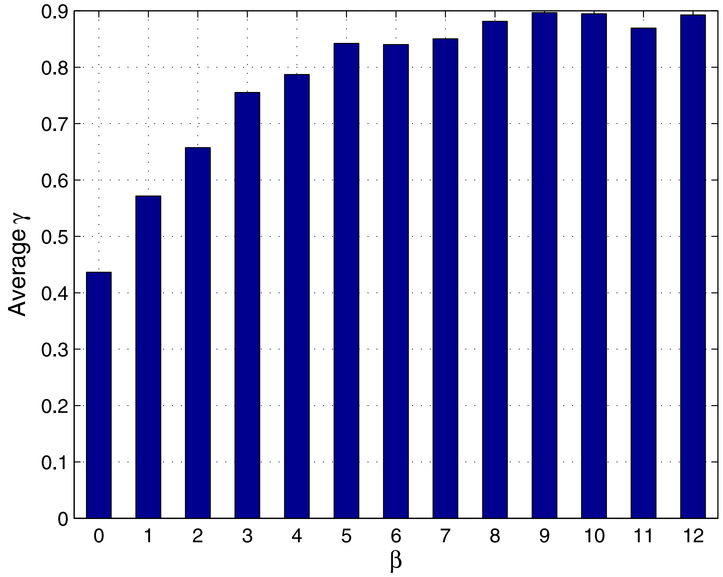


Fig. 10. Average degree of bi-polarization with different β with different β.

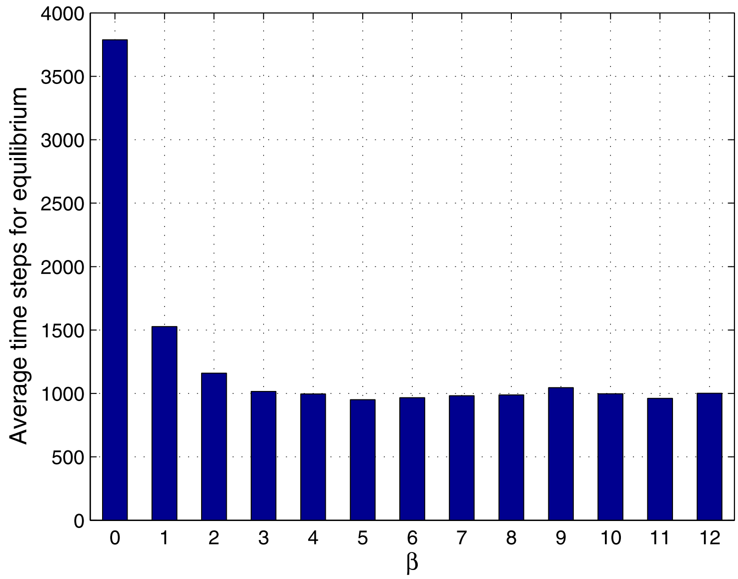


Fig. 11. Average time steps for all of the runs to reach equilibrium with different β.

opinion bi-polarization. When p is large and close to 1, the network is similar to random network, where the path length L is small and the agents are more connected to each other, thus it is more difficult for the population to lead to bi-polarization. In terms of network homophily [17], it can be said that the network with smaller p has higher degree of network homophily, vice versa.

Fig. 13 is the average γ as a function of p. It can be seen that $\overline{\gamma}$ remains almost unchanged when p increases from 10^{-4} to about 0.05 in this experiment, and then $\overline{\gamma}$ decreases rapidly when p increases from about 0.05 to 1. This trend is very similar to that in Fig. 12. It indicates furthermore large p can prevent opinion bi-polarization. The average convergence time steps taken by the population to reach equilibrium with different p are shown in Fig. 14. It can be observed that larger p can accelerate convergence. That is because the path length L of the network is small with large p.

Secondly, the result of the influence of the average degree of small world network on the opinion dynamics is shown in Figs. 15–17, where $n = 100,\ p = 0.1,\ \beta = 0$. The proportion of runs that end in bi-polarization with different $\langle K\rangle$ is shown in Fig. 15, which indicates that the proportion decreases rapidly as $\langle K\rangle$ increases from 10 to 50 in this experiment and then keeps almost unchanged as $\langle K\rangle$ continues to increase from 50. This can be explained as follows. The individuals in the small

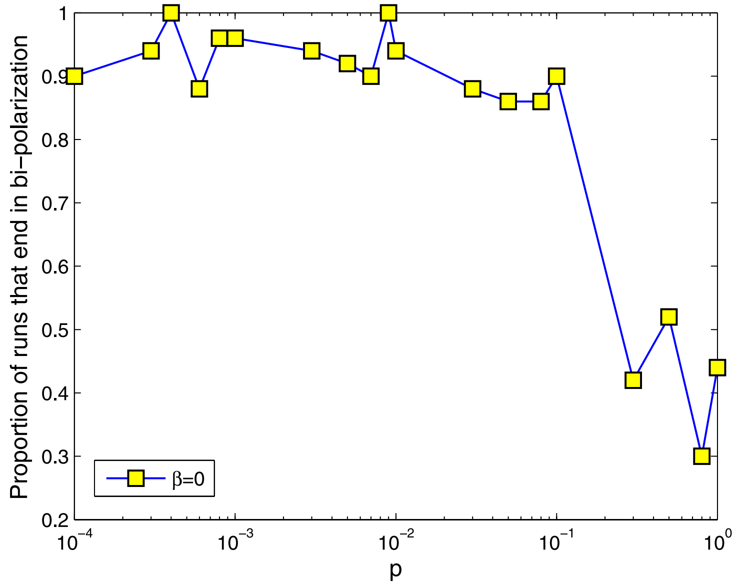


Fig. 12. Proportion of runs that end in opinion bi-polarization with different rewiring probability *p*.

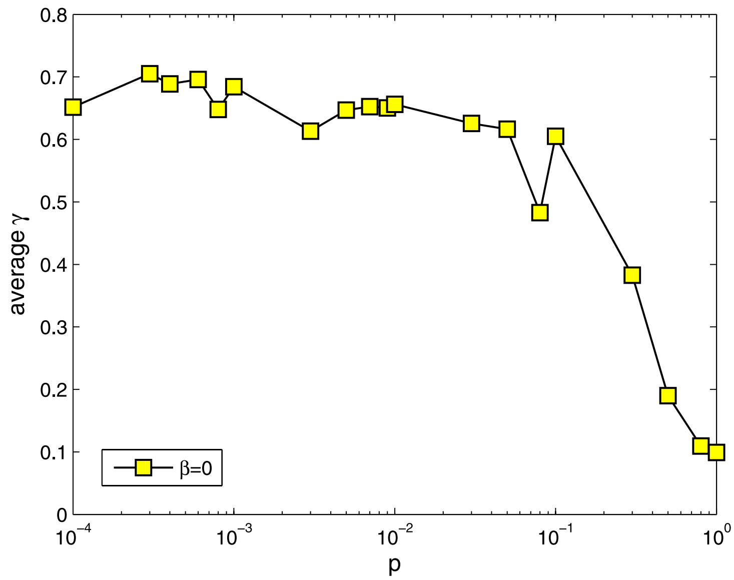


Fig. 13. Average degree of bi-polarization with different rewiring probability *p*.

world network are relatively isolated when $\langle K \rangle$ is 10, thus the final opinion of the population can lead to bi-polarization with large probability. As $\langle K \rangle$ increases, the network structure changes dramatically and the individuals becomes much less isolated. As a result, the proportion of the runs ending in bi-polarization drops sharply. But when $\langle K \rangle$ is large enough, even if $\langle K \rangle$ continues to increase, the network structure remains practically unchanged and is very similar to complete connected network, thus the proportion is very small and keeps almost unchanged, as shown in Fig. 15.

Fig. 16 presents the average degree of bi-polarization with different $\langle K \rangle$. It can be seen that $\overline{\gamma}$ drops rapidly for intermediate value of $\langle K \rangle$, and keeps almost constant for small and large value of $\langle K \rangle$. This can furthermore verify the conclusion that large $\langle K \rangle$ can contribute to opinion consensus.

Fig. 17 demonstrates the influence of $\langle K \rangle$ on the convergence speed of the opinion evolution. It can be seen that the average time steps for the population to reach equilibrium decreases as $\langle K \rangle$ increases. This is because as $\langle K \rangle$ increases, the individuals on the network are more connected with each other. The results indicate that large $\langle K \rangle$ can accelerate convergence.

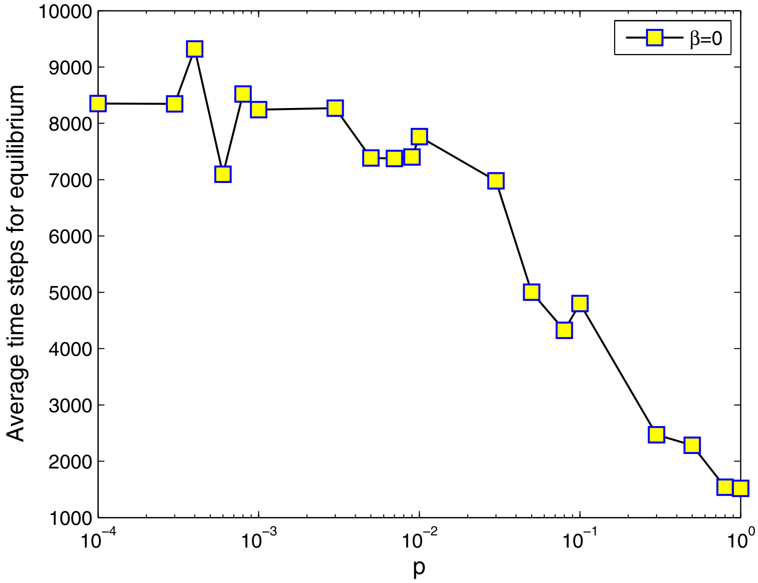


Fig. 14. Average time steps for all of the runs to reach equilibrium with different rewiring probability p.

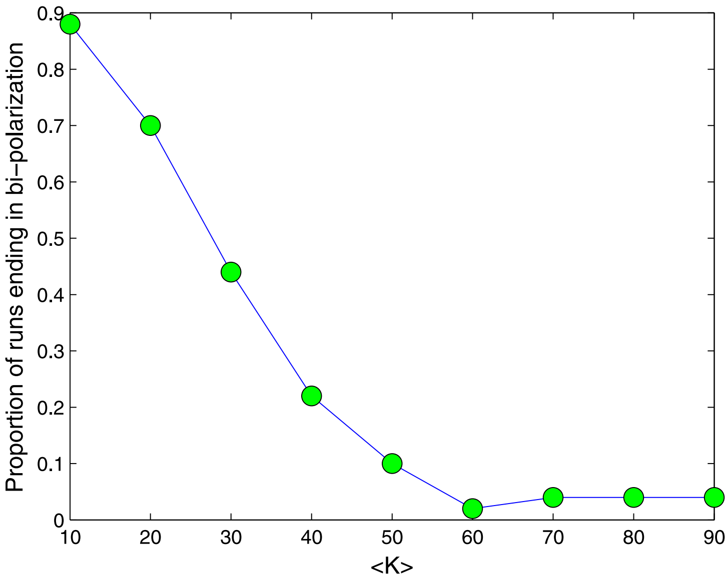


Fig. 15. Proportion of runs that end in opinion bi-polarization with different $\langle K\rangle$.

5. Conclusions

An agent-based model combining the effect of biased assimilation and homophily is proposed to investigate opinion bi-polarization. Based on persuasive argument theory, each agents opinion is represented by a finite number of arguments, and it communicates with its interactive partner by exchanging an argument chosen from its own argument set. Each agent chooses its interactive partner with a probability depending on their opinion distance during each interaction and considers its partner's opinion in a biased manner, which is described by a modified Urn process. The strength of homophily is reflected by the parameter β in Eq. (2). The proposed model surely results in either completely consensus in the extreme opinion or bi-polarization. Numerical simulations have been carried out on different network topologies to investigate the influence of biased assimilation, homophily and network structure on opinion formation.

It is shown that both biased assimilation and homophily indeed contribute to opinion bi-polarization and biased assimilation can also speed up convergence. Homophily plays more important part on the opinion bi-polarization compared to that of biased assimilation. Usually the stronger homophily, the higher probability to lead to opinion bi-polarization.

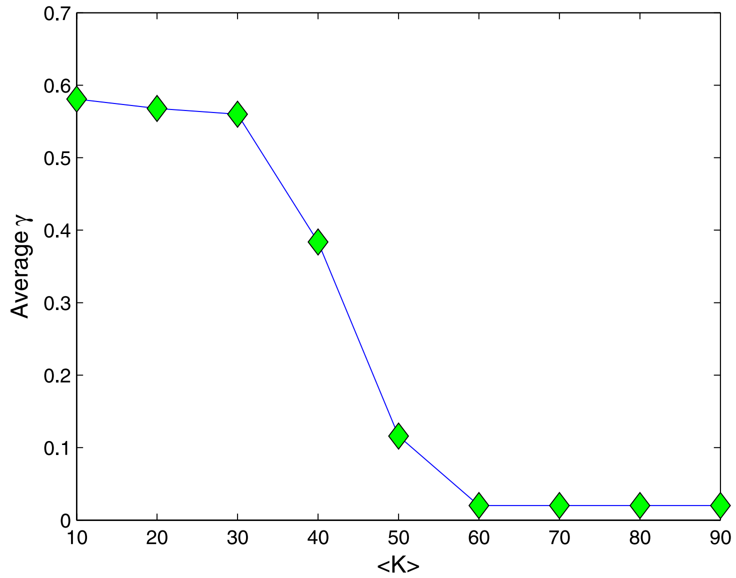


Fig. 16. Average degree of bi-polarization with different $\langle K \rangle$.

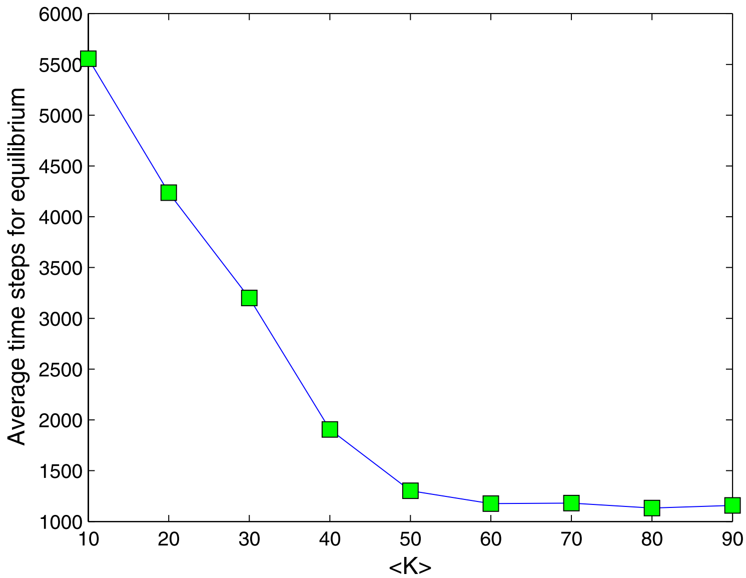


Fig. 17. Average time steps for all of the runs to reach equilibrium with different $\langle K \rangle$.

Stronger homophily can also enhance the degree of bi-polarization, and there is a threshold for this effect. But it seems to have little effect on the convergence time for the proposed model when $p_b = 1$, which is quite different from the result in Ref. [11]. It is also found that on complete network homophily plays a necessary role on opinion bi-polarization, which is almost impossible to achieve when the strength of homophily is zero, while on small network and karate network opinion bi-polarization can be easily achieved even when the strength of homophily is 0. Moreover, on small world network large rewiring probability p or large average degree $\langle K \rangle$ can contribute to opinion consensus and accelerate convergence. But only when p is large enough can the proportion of runs ending in bi-polarization drop rapidly and for small p the proportion keeps almost unchanged as p increases, while only when $\langle K \rangle$ is small enough can the proportion decrease rapidly and for large $\langle K \rangle$ the proportion remains unchanged. The influence of p and $\langle K \rangle$ on the convergence time steps exhibits similar trend.

The proposed model provides another insight for the study of opinion bi-polarization, which was mainly explained by negative influence or by homophily in Ref. [11]. Further work can be devoted to investigating the opinion dynamics based on the proposed model with moving agents.

Acknowledgments

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