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Analysis of naming game over networks in the presence of memory loss

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HIGHLIGHTS

- A modified naming game with memory loss is proposed.
- The strength of memory loss has little effect on the maximum number of different words.
- The maximum number of different words grows almost linearly with population size and coincides with each other under different strength of memory loss.

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ABSTRACT

In this paper, we study the dynamics of naming game where individuals are under the influence of memory loss. An extended naming game incorporating memory loss is proposed. Different from the existing naming game models, the individual in the proposed model would forget some words with a probability in his memory during interaction and keep his conveyed word unchanged until he reaches a local agreement. We analyze the dynamics of the proposed model through extensive and comprehensive simulations, where four typical networks with different configuration are employed. The influence of memory loss as well as the population size on the performance of the proposed model is investigated. The simulation results show that (i) the stronger memory loss, the larger convergence time; (ii) as the strength of memory loss becomes stronger, maximum number of total words will decrease, while the maximum number of different words among the population remains almost unchanged; (iii) the maximum number of different words increases linearly with the increase of the population size and coincides with each other under different strength of memory loss. The findings in the proposed model may give an insight to understand better the influence of memory loss on the transient dynamics of language evolution and opinion formation over networks.

1. Introduction

Research on collective behavior in social system is of great interest in the past decades and becomes one of the hottest inter-disciplinary topics [1–3]. Various agent-based models, for example Sznajd model [4], Galam model [5], bounded confidence models [6], naming game [7], have been designed by researchers with different background to investigate the features of social dynamics. Here naming game (NG) is concerned, which is originally proposed in [8,7] to describe the evolution of linguistic convention. It is an agent-based model played by a population over complete network, where

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individual interacts with each other through local peer-to-peer communication and the global consensus on the name assigned to the same object will be occurred eventually without a central coordinator. NG can be used to analyze the behavior of consensus problem in many other dynamical systems, such as opinion formation or negotiation [9,10], community formation [11].

NG has recently attracted lots of attentions, and researchers proposed various modified NG models from different aspects to further investigate the dynamics of language evolution and opinion formation. [12] proposed a generalized two-word NG model where in the case of successful interaction the individuals update their inventories with probability β, and found that the proposed model can display non-equilibrium phase transition. Lu et al. [13] gave broadcasting version of NG model to make it applicable in sensor network. Baronchelli [14] further investigated the dynamics of broadcasting NG model in terms of convergence time by considering only speaker or hearer updates its inventory respectively after a successful interaction and showed that the broadcasting scheme can contribute to a fast convergence. Li et al. [15] proposed a modified naming game where the speaker can speak to multiple hearers at one time and showed that the proposed model can reach convergence much faster than the minimal NG model, but decrease the individual's ability to learn new words. [16] studied the dynamics of NG with individual's preference. [17] considered the influence of propensity and stickiness on the dynamics of the two-word NG model. [18] considered the scenario where every agent played the role of both speaker and hearer at the same time, and proposed an extended NG model that considered the agents communicate with each other in groups and each speaker can convey multiple words at one time, which can accelerate the convergent speed compared to the model presented in [15]. Another modified NG model was proposed in [19] by considering the fact that the hearer will receive the wrong word with an error rate and the result indicated that the maximum number of different words increases linearly as the error rate increases.

All the aforementioned investigations on NG assumed that every individual in the population can exactly remember all the names in their memories, however, in the real world, people will inevitably forget something due to memory loss. Up to now, to the best of our knowledge there are few paper available studying the influence of memory loss on NG. Thus it is necessary to take into account the scenario that individuals are affected by memory loss. To this end, a new model of NG with memory loss (NGML) is proposed in this paper. In NGML, individual will drop some names in his memory, which apparently makes it more difficult to reach local agreement during the interaction. This paper aims to investigate how memory loss affects the performance of NG evolution over different network topologies, and attempts to give new insights into the evolution of language and opinion formation through local pairwise interaction.

The rest of this paper is organized as follows. Naming game with memory loss (NGML) is introduced in the following section, while the main results is presented in Section 3, devoted to analyzing the influence of memory loss on the evolutional dynamics of NGML on different network topologies. Finally some conclusions are given in the Section 4.

2. Model formulation

2.1. Naming game with memory loss

In the general NG, individuals start with empty memories and update their memories through pairwise interactions. At each iteration, one individual is chosen at random from the population to play the role of "speaker", while the "speaker" randomly selects one of his neighbors to be "hearer" and conveys one of his words in his memory to the hearer (if the speaker's memory is empty, he will randomly pick up one new word from the external vocabularies). The speaker and hearer update their memories based on the following simple rules: (i) the hearer compares the conveyed word from the speaker with his own memory. If the hearer has the same word, then such interaction is regarded as *success*, and both the speaker and the hearer will keep only the conveyed word and discard all other words, i.e. the speaker and the hearer reach a local agreement. (ii) otherwise, the interaction is regarded as *failure*, where the speaker will keep his memory unchanged while the hearer will add the conveyed word to his memory.

The interaction in NG is illustrated by an example shown in Fig. 1, where the word with gray background, "Blue", is the word conveyed by the speaker. As is shown in Fig. 1(a), the hearer does not have the word "Blue" in his memory, thus the interaction is regarded as *failure*. The hearer adds the word "Blue" into his memory while the speaker keeps his memory unchanged. Conversely, in Fig. 1(b) since there is the word "Blue" in the hearer's memory, the interaction is a *success*. Both the speaker and the hearer discard all words but "Blue" in their memories.

As is described in the previous section, it is assumed in NG that individuals always remember all the information well. However, memory loss is a very common phenomenon in the real world. It is inevitable that people forget some information stored in their memories. We will thus propose the model of naming game with memory loss (NGML) by modifying the updating rule of NG model. To take into account the influence of memory loss, we assume that each individual will discard some words in his memory at each time step. The number of missing words depends on the strength of the individual's memory loss. The main difference between NG and NGML lies in that if the forgotten word by the hearer happens to the word conveyed by the speaker, the interaction is then a failure, while in NG, such interaction would be success. An example in the Fig. 2 is taken to illustrate the difference. Due to memory loss, assume that the speaker conveys "Blue" to the hearer. As is shown in Fig. 2(a), if the hearer forgets the word "Blue" in his memory, the interaction is a failure, while this interaction would have been a success without memory loss. In this case, the hearer will add "Blue" into his memory after this interaction. Meanwhile, as shown in Fig. 2(b), if the hearer forgets the word "Yellow" rather than "Blue", this interaction

Speaker	Hearer
Red	Green
Blue	Yellow
Purple	

⟹

Speaker	Hearer
Red	Green
Blue	Yellow
Purple	Blue

(a) Failure.

Speaker	Hearer
Red	Green
Blue	Blue
Purple	

⟹

Speaker	Hearer
Blue	Blue

(b) Success.

Fig. 1. The interaction of minimal naming game. The word with gray background in (a)–(b) is the transmitted word of the speaker.

Speaker	Hearer
Red	Green
Blue	Yellow
Purple	~~Blue~~

⟹

Speaker	Hearer
Red	Green
Blue	Yellow
Purple	Blue

(a) Failure with memory loss.

Speaker	Hearer
Red	Green
Blue	~~Yellow~~
Purple	Blue

⟹

Speaker	Hearer
Blue	Blue

(b) Success with memory loss.

Fig. 2. The interaction of naming game with memory loss. The word with gray background is the transmitted word of the speaker, while the word with strikeout is the missing word resulting from memory loss of the hearer.

is a success and both the speaker and the hearer will discard all the words but the conveyed word "Blue" after the interaction, which is the same result as in NG.

It is worth to point out it is assumed in NGML only the hearer loses words during the interaction. We think this assumption is reasonable, because during the interaction, once the speaker is determined, the word received by the hearer is confirmed too and memory loss has no effect on the word conveyed by the speaker in such interaction. The following study is conducted under this assumption.

2.2. NGML on connected network

We will next the propose NGML over network in detail. We consider a community composed of n individuals denoted by $V = \{1, 2, 3, \ldots, n\}$. The communicating network is represented by $G = (V, E)$, where $E \subset V \times V$ is the set of edges and G has no self-loops, i.e. the individual cannot speak to himself. Each individual $i \in V$ has a memory that can store an arbitrarily large number of words. The state of individual i at time t is characterized by the tuple $(M_i, x_i(t))$, where M_i is the memory of individual i and $x_i(t) \in M_i$ is its conveyed word, respectively. Each individual starts with an empty memory. NGML ends when a consensus is reached, i.e. all individuals have only one and the same word in their memories.

For convenience, denote by N_i the set of neighbors of individual i, i.e. $N_i = \{j \in V \mid (i, j) \in E\}$. The network model of NGML is as follows:

step 1: A speaker $i \in V$ is randomly selected from the population. If $M_i(t) = \emptyset$, the speaker i will pick up one word at random from the vocabulary into his memory and assign this word as $x_i(t)$. Otherwise, i randomly selects one word from M_i as $x_i(t)$ (If the individual i has been a speaker before, he will keep his x_i unchanged).

step 2: The speaker i chooses one of its neighbors $j \in N_i$ at random as his hearer.
step 3: Due to the influence of memory loss, hearer j will forget some words in his memory $M_j(t)$, denoted by $M_j(t)^-$.
step 4: Hearer j checks whether his current memory contains $x_i(t)$. If $x_i(t) \subset \{M_j \setminus M_j^-\}$, such interaction is a *success* and both the speaker and hearer update their memories as $M_i = M_j = x_i(t)$; Otherwise, the interaction is a *failure*, and individual j will add $x_i(t)$ into his memory, and updates his memory as $M_j := \{M_j \setminus M_j^-, x_i(t)\}$, while the speaker i keeps his memory unchanged.
step 5: Repeat the above steps iteratively until the convergence state is reached.

Here we assume that during the interaction every individual will not forget his conveyed word $x_i(t)$, i.e. the set of forgetting words, $M_i^- \subset \{M_i \setminus x_i(t)\}$. Besides, each individual keeps its conveyed word unchanged during the interaction unless he reaches local agreement. This is different from NG, where the individual playing the role of "speaker" picks up at random a word from his memory to be his conveyed word in each interaction.

To measure the influence of memory loss, a parameter $p_f\,(0 < p_f < 1)$ is introduced, with which the individual $i \in V$ will forget the words in the set $\{M_i \setminus x_i(t)\}$. p_f can be individual-specific, but in order to simplify the problem, in this paper it is assumed to be the same for all the individuals. When $p_f = 0$, the NGML is reduced to original NG with unchanged conveyed word, while when $p_f = 1$, it means that the hearer will forgot all his words but his conveyed word. NGML will not reach global agreement when $p_f = 1$, thus in this paper we only consider the case when $p_f < 1$. More specifically, we focus on the scenario when $p_f \in [0, 0.5]$, which is much closer to the real world.

It is noted that the individuals V update their memories asynchronously and there are more than one pairwise interaction during every time step. For convenience to proceed the investigation on the dynamics of NGML over different network topologies, the following relevant quantities will be first introduced:

(1) $N_w(t)$: the number of total words defined as the sum of words held by all the individuals at time t;
(2) $N_d(t)$: the number of different words among all the individuals' memories at time t;
(3) $N_w^{\max}$: the maximum of the number of total words appearing throughout the process of evolution of NGML, i.e. the maximum value of $N_w(t)$; similarly, the maximum of the number of different words, $N_d^{\max}$, is the maximum value of $N_d(t)$ throughout the process of evolution of NGML;
(4) t_c: the convergence time defined as the time steps that are taken until the game ends;
(5) $S_r(t)$: the success rate, that is the average rate of success in each interaction during the time period t. We assume that during every time step, there are n interactions for the population with n individuals. If the number of successful interactions during time step t is m, the success rate can be calculated by

$$S_r(t) = \frac{m}{n}.$$

3. Results

In this section, the dynamics of NGML as well as the influence of network topology is analyzed, especially with respect to the relationship between the model parameters p_f and the relevant quantities $\{t_c, N_d^{\max}, N_w^{\max}\}$. The results are obtained by extensive numerical simulations.

3.1. Simulation setup

Simulations of NGML will be carried out on different networks: complete graph (CG), Random graph (RG) [20], small world (SW) network [21], and scale-free (SF) network [22]. We here employ 7 different networks, which are generated by altering the parameters of these models. The detailed properties of these networks are presented in Table 1. Here we consider the number of the individuals in the population $n = 1000$ in the following simulation results except the simulation about the influence of population size. The method to implement the simulation is given in the Appendix. The following results are an average over 50 independent realizations.

3.2. Evolution of the number of total words and different words

Fig. 3 shows the evolution of the total number of words N_w on different network topologies with different strength of memory loss. It can be seen that N_w increases much more sharply with smaller p_f at the beginning and then goes down to the stable value 1000, i.e. global consensus is reached, as time goes. Moreover, it can be also observed that N_w varies similarly with different p_f on the networks of RG/0.01, SW/10/0.03, SW/10/0.3, and SF/3/2, while the evolution differs much more under different p_f on other networks. That may be due to the fact that these networks are much more sparse compared to other networks.

The evolution of number of different words N_d is presented in Fig. 4, from which it can be seen that generally N_d reaches its plateau quickly at the beginning and then go downs to the stable value. It is also found that the variation of N_d exhibits obvious phase transition when p_f is small (in these simulations $p_f \leq 0.1$) while N_d varies smoothly when p_f is large on the dense network such as CG. That is due to the fact that when p_f is larger, more words in the individuals' memory will be

Table 1
Properties of networks with different configuration, where p is the probability with which any two nodes can be connected in random graph, p_r the rewiring probability of any nodes in small-world network, while n is the number of nodes, $\langle K \rangle$ is the average degree of the network, $\langle pl \rangle$ the average path length, and $\langle C \rangle$ the average coefficient of cluster.

Network configuration	Notation	n	$\langle K \rangle$	$\langle pl \rangle$	$\langle C \rangle$
CG	CG (1000)	1000	999	1	1
RG with $p = 0.01$	RG/0.01	1000	10.0640	3.2455	0.0098
RG with $p = 0.2$	RG/0.2	1000	198.774	1.8010	0.199
SW with $k_0 = 10, p_r = 0.03$	SW/10/0.03	1000	20	3.9349	0.6508
SW with $k_0 = 10, p_r = 0.3$	SW/10/0.3	1000	20	2.8115	0.2671
SW with $k_0 = 20, p_r = 0.3$	SW/20/0.3	1000	40	2.3543	0.2676
SF with $n_0 = 3, m = 2$	SF/3/2	1000	3.9480	4.0750	0.0397
SF with $n_0 = 30, m = 29$	SF/30/29	1000	53.9720	2.0204	0.1167

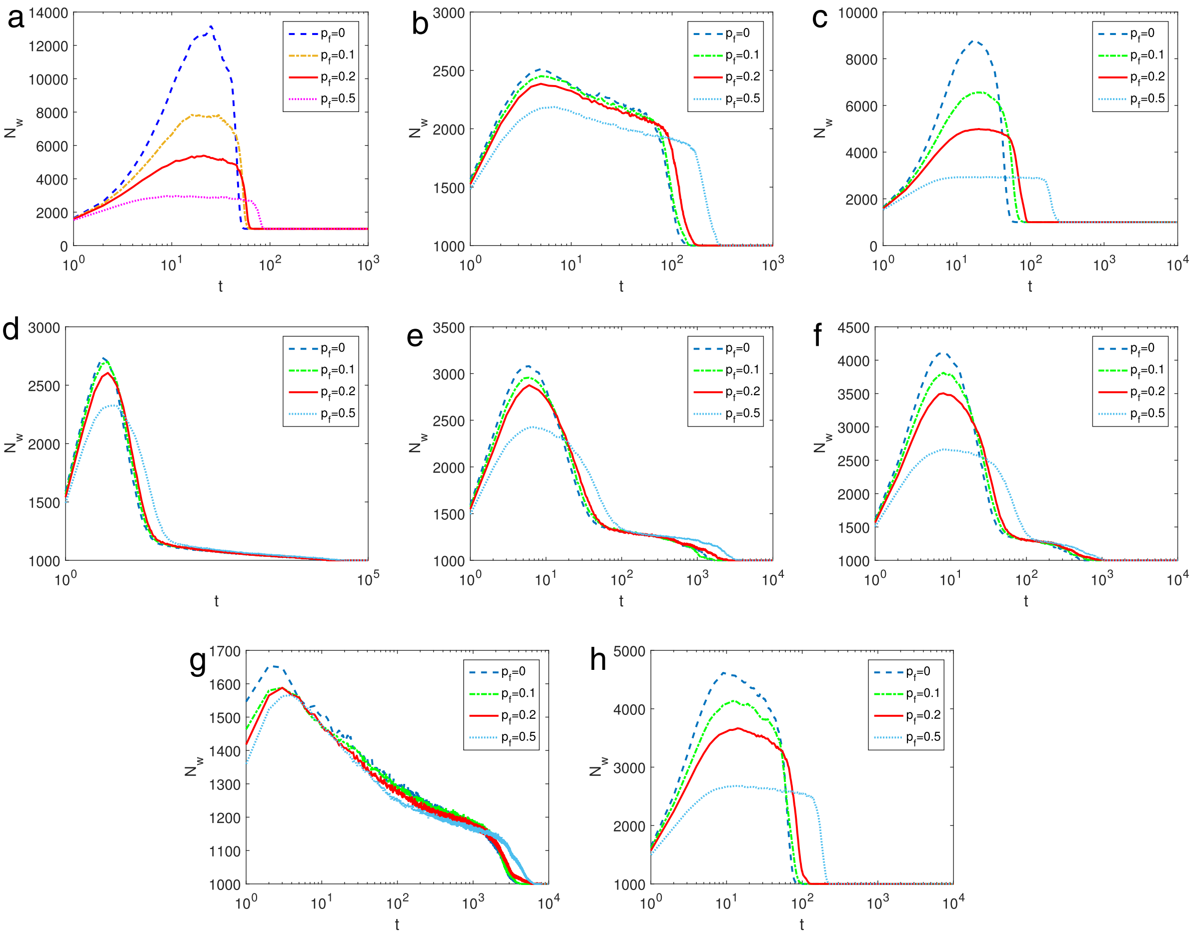

Fig. 3. Time evolution of the number of total words on different network topologies, (a) CG, (b) RG/0.01, (c) RG/0.2, (d) SW/10/0.03, (e) SW/10/0.3, (f) SW/20/0.3, (g) SF/3/2, (h) SF/30/29.

lost. In addition, it can be observed that the sparser the network topology, the less different the evolution of variation of N_d among different strength of memory loss. For example, the variation of the evolution of N_d is very similar with different p_f on SF/3/2, as is shown in Fig. 4(g).

The according success rate s_r is presented in Fig. 5, from which it can be seen that s_r exhibits obvious phase transition with different strength of memory loss on dense networks, such as CG, RG/0.2, SF/30/29, while it increases much more smoothly on other networks. Moreover, it is also found that the trend of the evolution of s_r is very similar with different p_f on all of these networks. This indicates that the influence of the strength of memory loss on the trend of success rate is very minor.

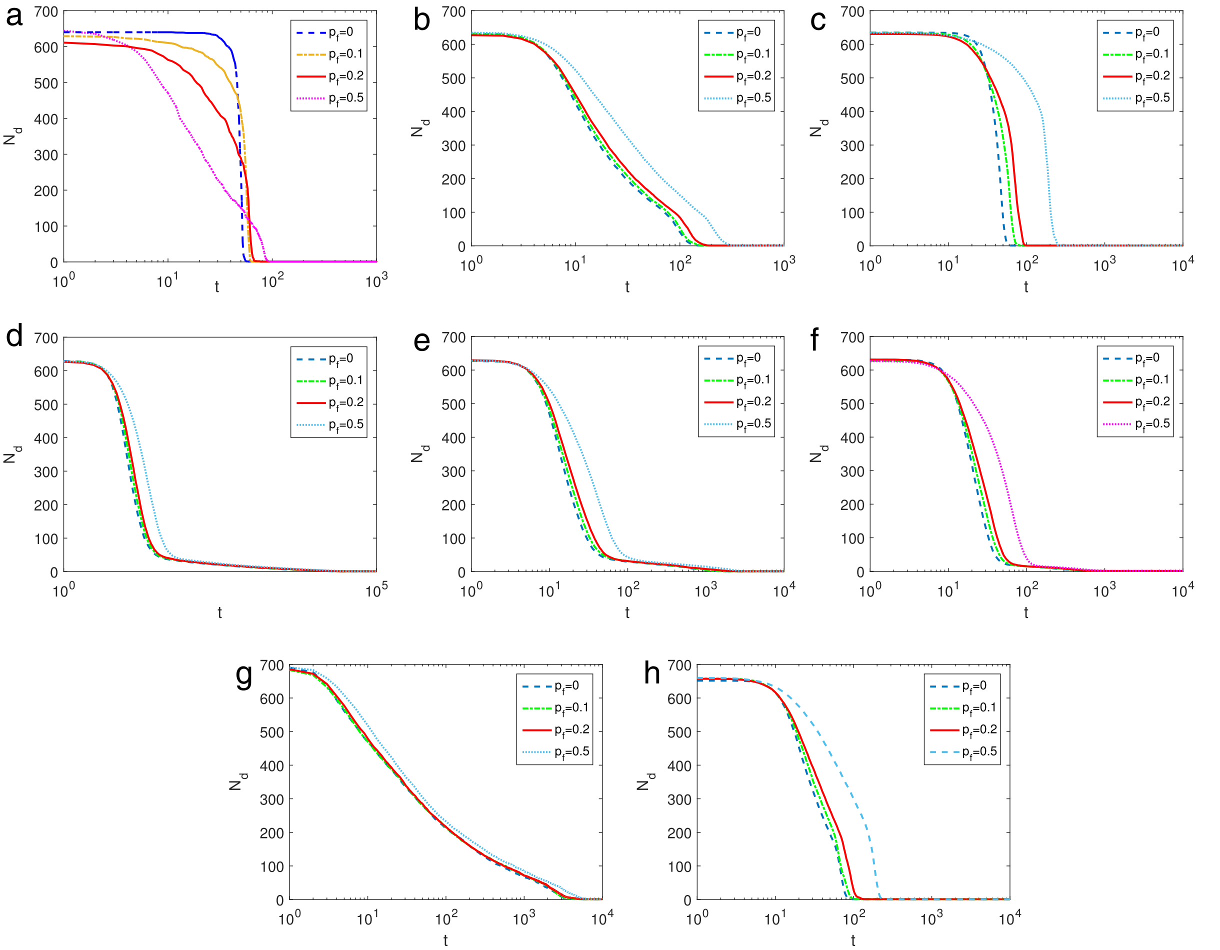

Fig. 4. Time evolution of the number of different words on different network topologies, (a) CG, (b) RG/0.01, (c) RG/0.2, (d) SW/10/0.03, (e) SW/10/0.3, (f) SW/20/0.3, (g) SF/3/2, (h) SF/30/29.

3.3. Relationship between memory loss and the maximum number of total words and different words

Inspired by the work in [19], we also concern the influence of the strength of memory loss p_f on the maximum number of total words and different words. Here we consider the case when p_f varies from 0 to 0.5.

Fig. 6 is the relationship between the maximum number of total words $N_w^{\max}$ and the strength of memory loss p_f. It can be seen that intuitively the maximum number of total words $N_w^{\max}$ goes down as p_f becomes larger. That is because the hearer will forget more words with larger p_f. The number of total number of words will thus decline accordingly.

The variation of $N_d^{\max}$ with different p_f is presented in Fig. 7, from which it can be observed that $N_d^{\max}$ keeps almost unchanged with different p_f on these networks. It can thus indicate that the influence of memory loss on $N_d^{\max}$ is very minor. It is also interesting to find that the difference of $N_d^{\max}$ on different network topologies is very small too. For example, the evolution of $N_d^{\max}$ with different p_f on the networks shown in Table 1 is very close to each other. Thus it can be conjectured that neither the strength of memory loss nor the structure of network will influence the maximum number of different words.

3.4. Relationship between memory loss and convergence speed

The convergence time t_c of NGML on different network topologies with different p_f is presented in Fig. 8, from which it can be seen obviously that larger p_f will lead to longer time steps required to reach consensus for the population in NGML. This is intuitive. The larger p_f, the more words lost. As a result, the probability for the interactive individuals to reach local agreement drops off, which leads to larger convergence time.

From Fig. 8 it can be also found that t_c may be quite different from different network topologies. For example t_c on the network of SW/10/0.03 and SF/3/2, shown in Fig. 8(d) and (g), is much larger than those on other networks. However, t_c with

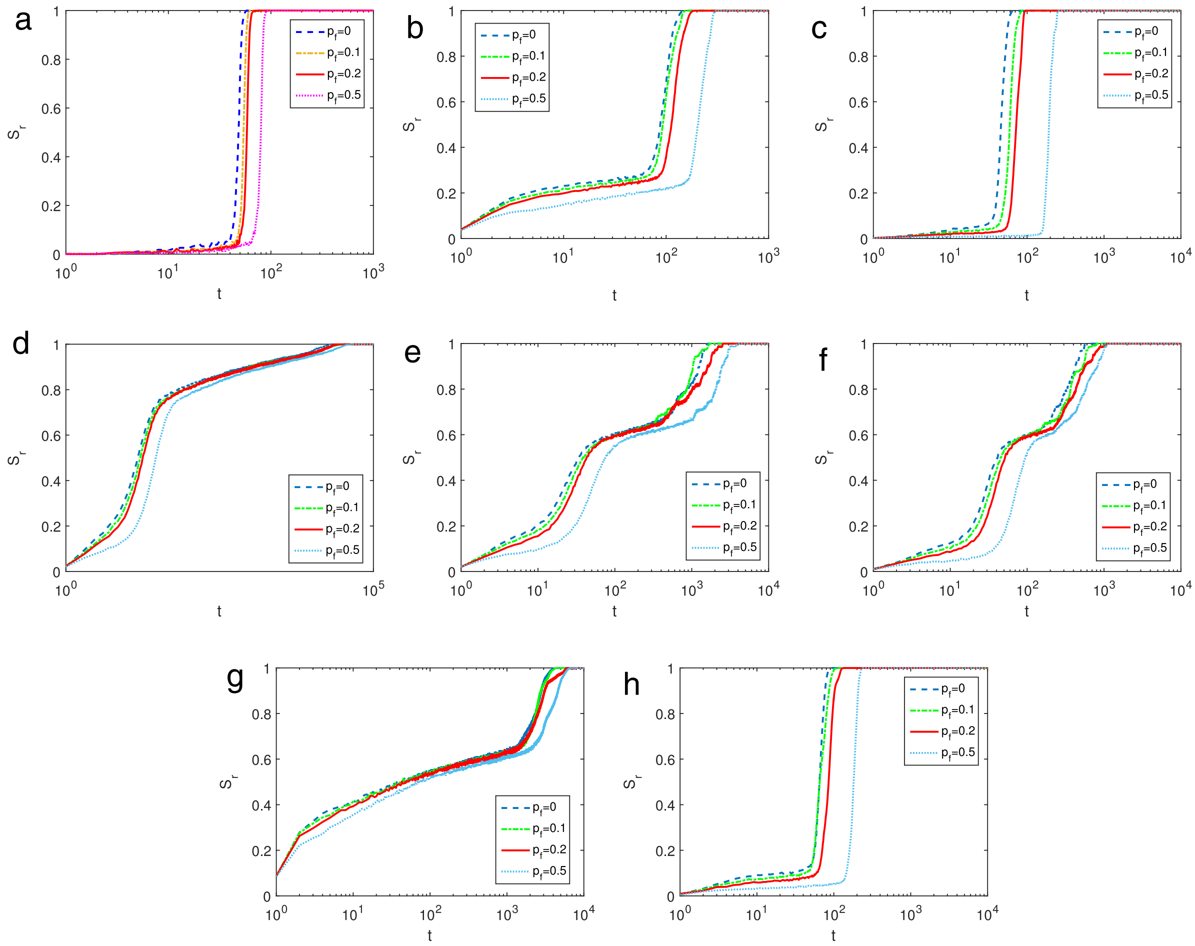

Fig. 5. The variation of success rate with different p_f on different network topologies, (a) CG, (b) RG/0.01, (c) RG/0.2, (d) SW/10/0.03, (e) SW/10/0.3, (f) SW/20/0.3, (g) SF/3/2, (h) SF/30/29.

different p_f on the same network topology is within the same order. For example, the order of t_c is 10^4 as p_f varies from 0 to 0.5 on the network of SW/10/0.03, while it is 10^2 on the network of SW/20/0.3. These results indicate that although larger strength of memory loss can increase the convergence time, network topology may play a more important role on the value of t_c.

3.5. *Effect of population size n*

We next investigate the impact of population size n on the dynamics of NGML, especially concerning the influence of n on convergence time, the maximum number of total words and different words. The population size n is assumed to increase from 100 to 2000. Here only the results on complete networks are presented.

Fig. 9 is the evolution of the number of total words N_w, different words N_d, and the success rate s_r, respectively, where the strength of memory loss $p_f = 0.2$. Actually we also carried out simulations with different p_f and found the evolution of N_w, N_d and s_r exhibits similar shape. It can be seen from Fig. 9 that the trend of variation of N_w, N_d, and s_r is very similar with different population size n and all of them exhibit obvious phase transition. This indicates that the population size has little impact on the dynamical shape of N_w, N_d and s_r.

The evolution of maximum number of total words $N_w^{\max}$, different words $N_d^{\max}$, and convergence time t_c with different population size n are presented in Fig. 10. Here in order to demonstrate the influence of memory loss, the results of both $p_f = 0$ and $p_f = 0.2$ are presented. It can be found from Fig. 10(b) that t_c, $N_w^{\max}$ and $N_d^{\max}$ are proportional to the population size n, especially $N_d^{\max}$ which increases almost linearly with the population size n. It is also surprising to find that the evolution of $N_d^{\max}$ with $p_f = 0$ almost coincides that with $p_f = 0.2$, which furthermore confirms that memory loss has little impact on the maximum number of different words.

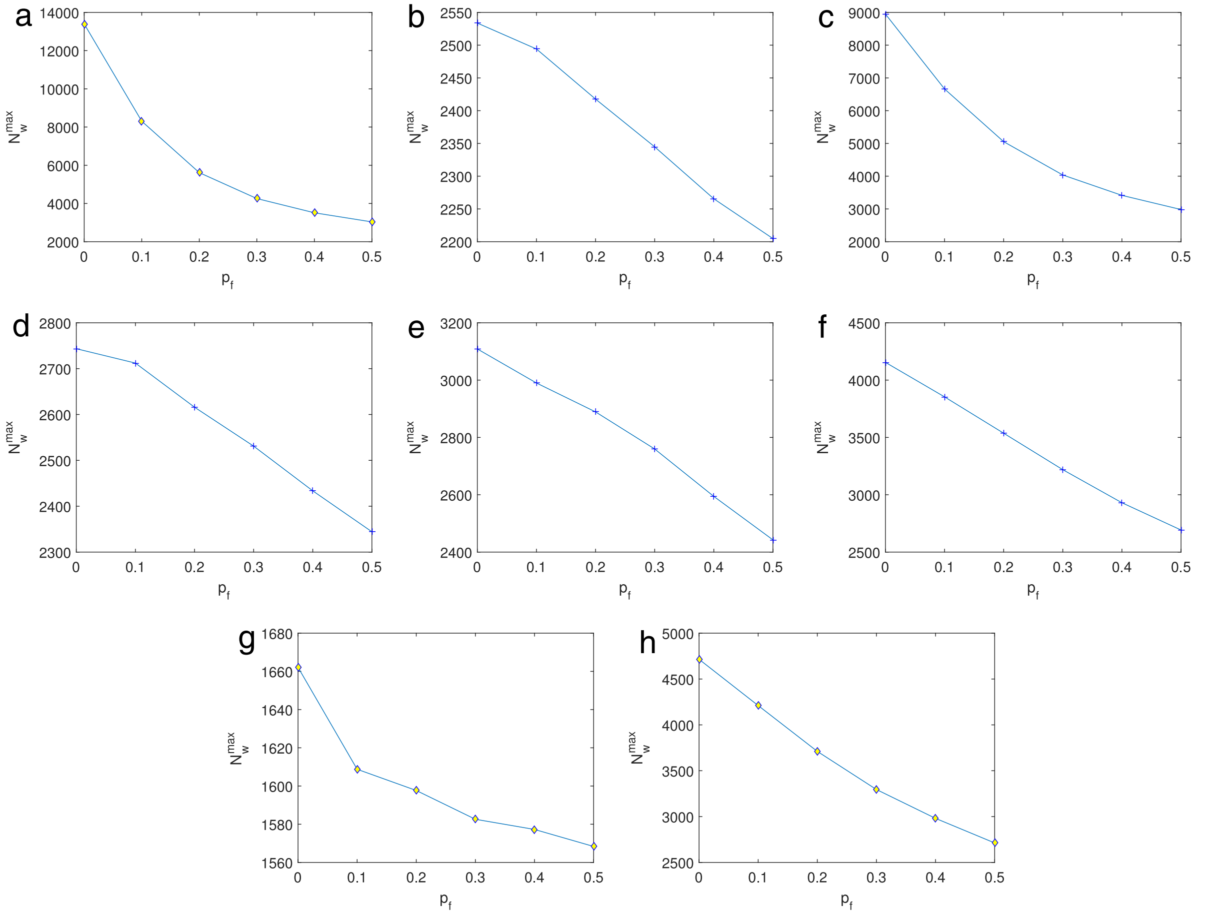

Fig. 6. The average maximum number of total words with different p_f on different networks. (a) CG, (b) RG/0.01, (c) RG/0.2, (d) SW/10/0.03, (e) SW/10/0.3, (f) SW/20/0.3, (g) SF/3/2, (h) SF/30/29.

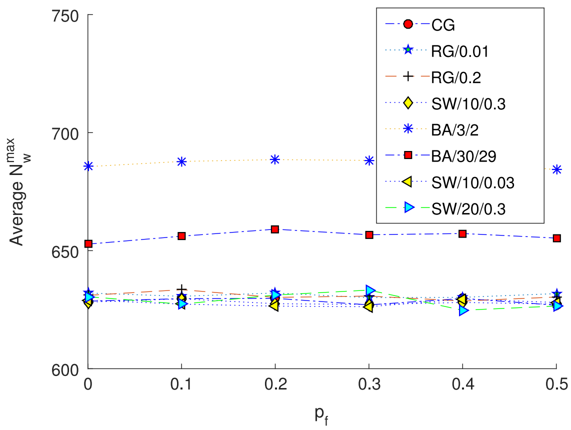

Fig. 7. The average maximum number of different words with different p_f on different network topologies.

4. Discussion and conclusion

In this paper, an extended naming game model incorporating the influence of memory loss is proposed. Different from the traditional NG model, in NGML individual may forget some words in his memory with a certain probability, and always keeps his conveyed word unchanged unless he reaches a local agreement. The dynamics of NGML is studied through extensive and comprehensive computer simulations. Specifically, we evaluated the effects of memory loss as well as population size on the performance of NGML on different network topologies.

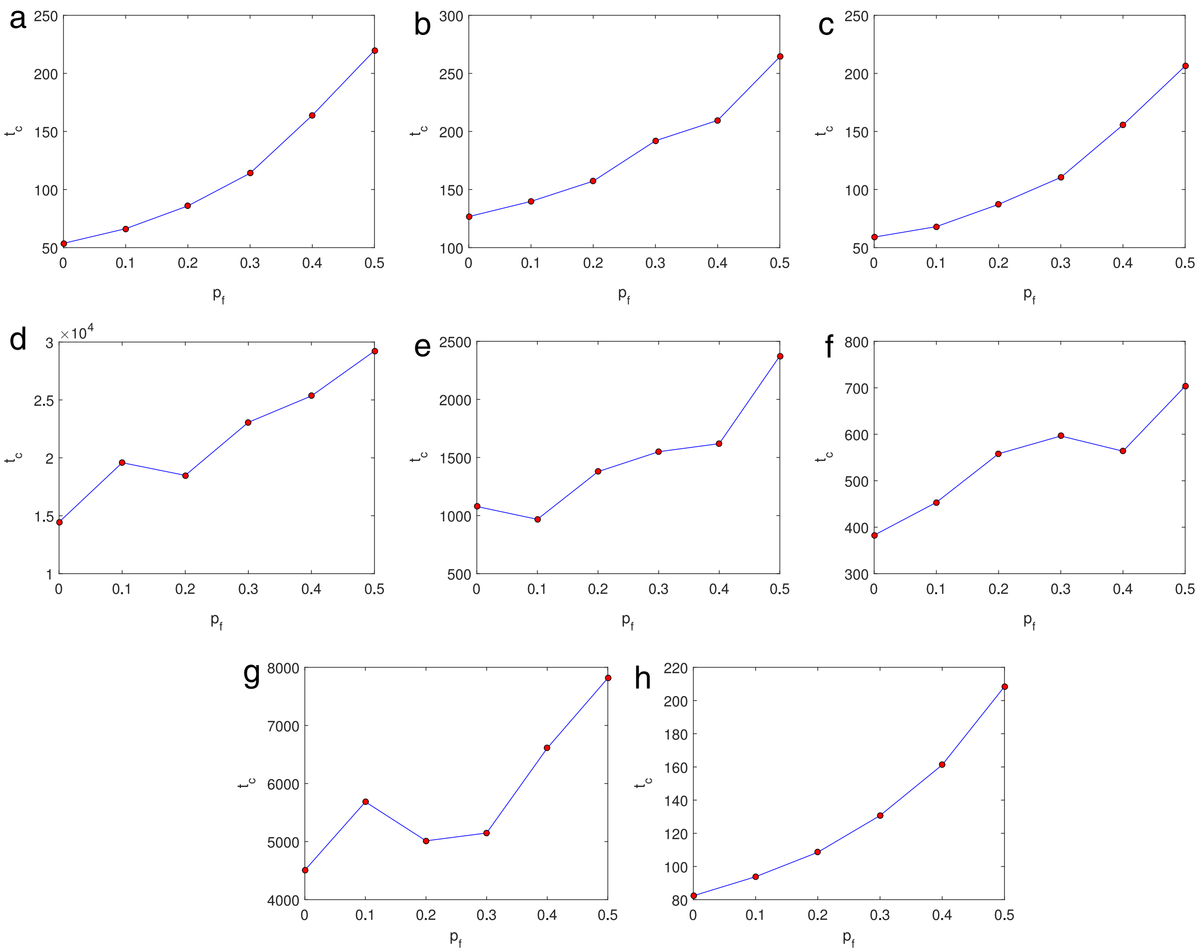

Fig. 8. Average convergence time for NGML with different p_f on different network topologies, (a) CG, (b) RG/0.01, (c) RG/0.2, (d) SW/10/0.03, (e) SW/10/0.3, (f) SW/20/0.3, (g) SF/3/2, (h) SF/30/29.

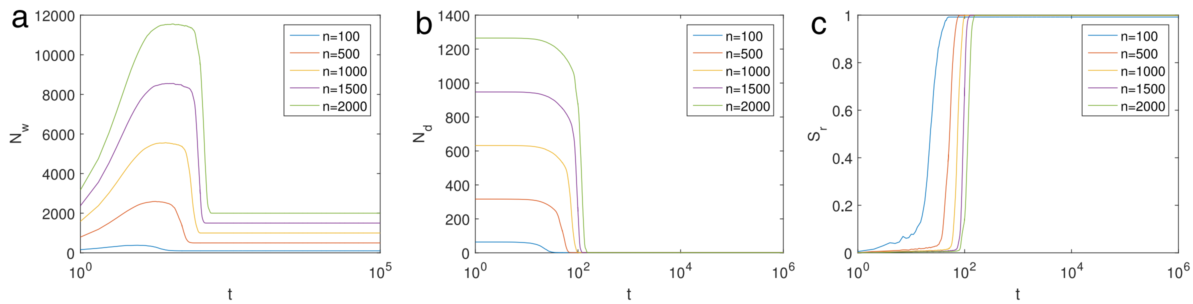

Fig. 9. Variation of dynamics of NGML over complete network where $p_f = 0.2$. (a) the total number of the words, (b) the number of different words, (c) the success rate.

It is found that the population on the connected network will eventually reach global agreement, as long as the strength of memory loss p_f is not very large (here p_f is considered to be within [0, 0.5]). Moreover, memory loss has little influence on the shape of evolution of success rate, while larger strength of memory loss would make the evolution of total words and different words vary smoothly on the dense networks. In addition, intuitively stronger memory loss can prolong the time for the population to reach convergence and decrease the maximum total number of words among the population, and larger population size will increase the convergence time as well. The results also show that convergence time may depend more on the network topology when the strength of memory loss is not very large, i.e. the convergence time of the same population size is almost within the same order with different strength of memory loss on the same network topology, while the order would be quite different from each other on different network topology.

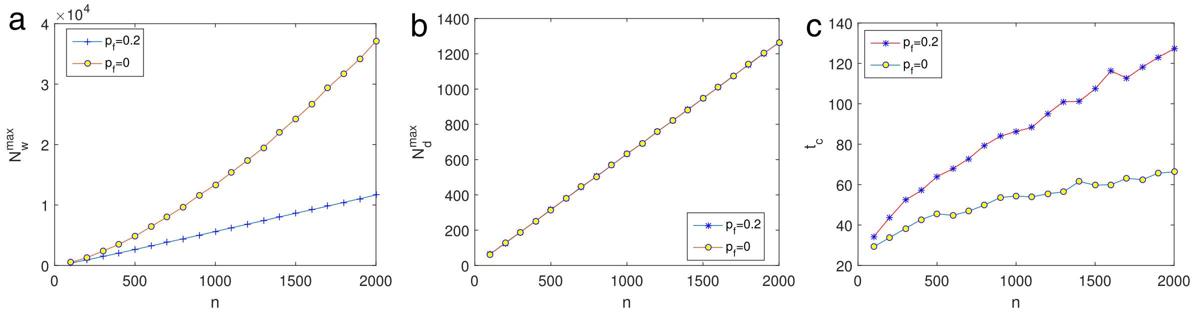

Fig. 10. The relationship between the population size n and (a) the average maximum number of total words $N_w^{\max}$, (b) the average maximum number of different words $N_d^{\max}$, (c) the average convergence time.

It is also interesting to observe that the maximum number of different words keeps almost unchanged with different strength of memory loss, and even differs very little on different network topologies. Moreover, the maximum number of different words increases almost linearly with population size and the curve almost coincides with each other under different strength of memory loss. These observations imply that small memory size can result in global consensus on the condition that the maximum level of diversity of keeps unchanged.

The findings reported can be helpful to understand better the influence of memory loss on naming game, moreover, they can shed lights on analyzing the evolution of language and opinion formation based on peer-to-peer communication. But here the contact network of the interactive population is assumed to be single and the communication between the interactive individual is free of the influence of external uncertainty. Thus future work can focus on the analysis of NGML with noise and the influence of multiplex network topology.

Acknowledgments

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Appendix. Method

The implementation of NGML over network will be presented in this appendix. In order to facilitate the algorithm to realize the simulation, several variables would be introduced at first.

Denote by l_i and l_j the number of words stored in speaker i and hearer j's memories, respectively.

In addition, for the same word k, it may be held by more than one individuals at the same time. Thus a variable $f(k)$ is introduced to record the number of times of the word k occurred in population at one time. If $f(k)$ reduces to zero, namely the word k does not appear in the memory of the population, then the number of different word n_d will be decreased by 1.

Denote as d the number of the words in of i and j's memory where $f(k)$ reduces to zero after the interaction.

As described before, there are 4 possible situations for any individual during each interaction. If the quantities $N_w(t)$ and $N_d(t)$ are determined, other relative quantities can be easily obtained. We thus concern the update rules of these two quantities in each situation, which are given as follows:

(1) *Invention*: this happens when the speaker i's memory is empty, then it will pick up one word (denoted by k) at random from the vocabulary. In this case, the quantities N_d, N_w will be updated as Eq. (A.1)

$$N_d = \begin{cases} N_d + 1, & \text{if } f(k) = 1 \\ N_d, & \text{if } f(k) > 1 \end{cases}$$
$$N_w = N_w + 1. \tag{A.1}$$

(2) *Memory loss*: In this case, the individual discards the words in his memory with a certain probability p_f. Let the number of the forgotten words M_j^- as l_f, and the number of the words in M_j^- whose $b(k) = 1$ as r_{fd}. The quantities N_d, N_w are updated by Eq. (A.2)

$$N_d = N_d - r_{fd}$$
$$N_w = N_w - l_f. \tag{A.2}$$

(3) *Failure*: If the hearer j does not yet have the conveyed word in its memory, then the hearer j will add the received word into its memory while the speaker i keeps its memory unchanged. In this case, the number of different words keeps

Algorithm 1 Naming game with memory loss

Input: $n, G(V, E), p_f, T_{stop}$;
Output: N_w, N_d, N_s, t_c.
repeat
  for $k = 1 : n$ **do**
    A speaker $i \in V$ is randomly chosen
    if M_i is empty **then**
      Individual i picks up one word at random from the vocabulary and add it into M_i.
      update N_d, N_w
    end if
    if x_i is NULL **then**
      i randomly select one word from M_i as x_i
    end if
    i picks up one of his neighbor j from N_i as his hearer.
    if j's memory M_j contains x_i **then**
      The interaction is success;
      $M_i = M_j = x_i$,
      update the parameters: $N_w(t), N_d(t), N_s(t), x_j, C$
    else
      The interaction is failure.
      for all word w_j in M_j **do**
        j forgets w_j with probability p_f.
        update $C(w_j)$
      end for
      j adds x_i into his current memory.
      update parameters: $C(x_i), N_w, N_d, N_w(t)$
    end if
  end for
until All of the individuals have only one same word, or the termination time is reached.

unchanged, while the number of total words N_w increased by 1. The update rule for N_w and N_d is presented in Eq. (A.3)

$$\begin{aligned} N_d &= N_d \\ N_w &= N_w + 1. \end{aligned} \tag{A.3}$$

(4) *Success*: In this case both the speaker i and the hearer j will keep only the conveyed word $x_i(t)$ and delete all the other words in their memories. Due to the fact that there may be some words that only existing in the memory of individual i and j's memories, these words will not appear in the memory of the population after the interaction. Let the number of such words as r_d. The concerned quantities N_d, N_w will be updated by Eq. (A.4).

$$\begin{aligned} N_d &= N_d - r_d \\ N_w &= N_w - l_i - l_j + 2. \end{aligned} \tag{A.4}$$

The algorithm to implement the evolution of NGML is presented as follows.

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